

# On the optimality of the patent renewal system

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*The patent system is mainly a renewal system: the patent life is chosen by the patentee in return for fees. I ask whether such a system can be justified by asymmetric information on costs and benefits of research. In such a model I show that renewal mechanisms (possibly with subsidies) are equivalent to direct revelation mechanisms and therefore cannot be improved on, regardless of the objective function. Under plausible circumstances, patents should have a uniform life, rather than varying in length, as typically occurs under a renewal system.*

## 1. Introduction

■ A standard justification for the patent system is that it delegates R&D decisions to firms. Firms have better information on the costs and benefits of R&D than the government has and can thus make better decisions. If the patent authorities were as well informed as firms, a better system would be to commission R&D directly. Firms could then be remunerated using optimal taxation rather than patents, which would reduce deadweight loss to consumers. In addition, profits could be held to a fair rate of return on the research investment. The patent system as now constituted gives rewards that do not depend on the R&D cost.

But even if firms have better information than the patent authorities, the patent system still seems to be a suboptimal incentive mechanism. An early critic was Wright (1983), who argued that other reward mechanisms could dominate. Among other things, patents lead to patent races, which increase R&D expenditures and can lead to duplication of R&D costs. Of course the increased expenditures can also accelerate innovation, which is an offsetting benefit (Loury, 1979).

Even if the benefits of acceleration dominate the costs of duplication, patent races do not use private information efficiently. On the cost side, there is no way for firms in the race to identify low-cost firms or delegate effort to them (Gandal and Scotchmer, 1993). On the benefit side, firms in a race may “herd” on an inefficient investment strategy because they make incorrect, but self-reinforcing, inferences from observing

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each other's investment decisions (Minehart and Scotchmer, 1999). Crémer and Scotchmer (1996) and Scotchmer (1999) address the problem of aggregating information by designing an optimal mechanism. They show that, due to correlation in the firms' private information on the patent's value, a mechanism designer can elicit the information on value costlessly. Consequently, firms need not be compensated more than their R&D costs, except to the extent that they earn an "informational rent" related to cost revelation. Those mechanisms solve the dual problems of implementing an efficient investment decision and delegating research effort to the lowest-cost firm, but they bear little resemblance to a patent system.

The literature on patent races is not optimistic about the efficiency of patents, so it is perhaps surprising that there is an economic environment in which the patent system turns out to be optimal. This occurs when only one firm has the "idea" for an innovation (O'Donoghue, Scotchmer, and Thisse, 1998) and must decide whether to invest in it. I show that in such an environment, if the mechanism's transfers cannot use *ex post* information on value or cost, the only feasible incentive mechanisms are equivalent to patent renewal systems, where fees can be positive or negative.

Patent renewals are a feature of European patent systems that was adopted in the United States in 1982. The patent holder can renew a patent up to some maximum life in return for fees that increase over time. See Lanjouw, Pakes, and Putnam (1996) for a recent survey of how economists have used this system to make inferences about the distribution of patent values, including Pakes and Schankerman (1984), Schankerman and Pakes (1986), Pakes (1986), Schankerman (1998), and Lanjouw (1998). The renewal system turns out to be very important in practice. A regularity across technology classes and countries is that no more than 50% of patents are maintained more than ten years (Lanjouw (1998) for Germany, Schankerman (1998) for France). Pakes (1986) reports that fewer than 7% of patents in France and 11% in Germany are renewed for full term. Schankerman (1998) reports that there is considerable variance in renewal rates if patents are categorized by technology and nationality of owner, but the proportion of patents renewed to full term is not higher than about 30% in any of the classes he investigates.

In the model presented in Section 2, an idea is defined by two unobservable parameters,<sup>1</sup> namely, a cost of R&D and a signal of the innovation's value, and the firm with the idea must decide whether to invest in it or discard it. Investment in the idea is efficient if the value is sufficiently higher than the cost. The patent system determines which ideas elicit investment, but since information is private, an optimal mechanism will typically not implement the first best.

Cornelli and Schankerman (1999) also examine a one-firm investment decision, but in a different model of asymmetric information. They assume that the firm has an unobservable one-dimensional productivity parameter that determines both the value of the innovation and the amount of money the firm will spend achieving it. Using the same techniques as below, the appendix to my (1997) working paper shows that renewal mechanisms are also equivalent to direct mechanisms in their model. Cornelli and Schankerman show how the renewal system can shift aggregate R&D effort toward higher-productivity firms, and they compare the optimal renewal fees to the ones in practice. Another article on patent renewals is Crampes and Langinier (1998), who

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<sup>1</sup> In most principal-agent problems, there is only one unobserved parameter. Exceptions are Lewis and Sappington (1988) and recent (independent) work by Rochet and Choné (1998), who study the problem of a monopolist choosing a menu of qualities for agents with multidimensional types. That problem has structure similar to the one here.

argue that patent terminations reveal private information on value, and firms might modify their decisions on whether to renew in order to disguise this information.

Sections 3 and 4 characterize what is implementable and show equivalence with renewal mechanisms. The optimality of renewal mechanisms might seem counter-intuitive. After the costs of R&D are already paid, the patentee faces a tax-and-transfer scheme whereby he pays for patent life and is reimbursed through inefficient monopoly pricing. Further, the highest-value innovations will be renewed longest, and this compounds their profit: not only are high-value innovations more profitable in each period, but they receive a longer patent. This seems inefficient unless the R&D cost of high-value innovations is proportionately higher than the cost of low-value innovations. However, the opposite might also hold: high-value innovations might be largely the product of serendipity rather than hard work, and one might therefore expect R&D cost to increase less than proportionately with the value of innovations.

This is precisely where the equivalence between direct revelation mechanisms and renewal mechanisms is informative. Suppose we index potential innovations by their per-period market value  $v$ . I show in Section 3 that if a decision rule is implementable, the maximum R&D cost that is supported for each value  $v$  is given by a “cutoff function” that must be convex. (This is  $\delta$  below.) Suppose, however, that the cost of R&D is a concave function of the value of the project. Then there is no feasible mechanism that exactly reimburses the firms their costs. The best mechanism will overreward some firms and cut off some innovations.

Section 5 introduces a notion of social welfare and characterizes efficient mechanisms. The optimal decision rule depends on the distribution of costs and benefits, and also on whether the patent system can overcome the problem of opportunistic firms and provide subsidies. It follows from the incentive constraints that subsidies, if allowed, will be kept by low-value innovators with no patents, and high-value innovators will return part of the subsidy (perhaps earning negative subsidies on net) in return for patents. I identify circumstances where a fixed patent life is optimal, with and without subsidies, and other circumstances where it is optimal to let the firm choose between a subsidy with no patent life and a smaller (possibly negative) subsidy plus a patent. These arguments buttress those of Shavell and van Ypersele (1998), who study a model of one-dimensional asymmetric information that is formally similar to that of Cornelli and Schankerman (1999) and show that subsidies in addition to (fixed-life) patents can be optimal.

Unfortunately I know of no evidence on the joint distribution of the value and cost of innovations. The empirical articles above, as well as a recent literature based on surveys (see Scherer, 1998) estimate the distribution of values, but there are no data on cost. The distribution of values is highly skewed, with many innovations being valueless and a handful having very high value.

## 2. Model

■ This section and the next describe a model of asymmetric information on costs and benefits of innovation and characterize the decision rules that can be implemented with incentive-compatible mechanisms. The main conclusion is that incentive-compatible mechanisms and renewal mechanisms are equivalent. Thus, whatever the objective function of the patent authority, and whatever the distribution of projects’ costs and benefits, the optimal mechanism will be a renewal mechanism. Nothing else is feasible. (A trivial renewal system is the fixed-patent-life system.)

The firm has R&D cost  $c$ , drawn from a distribution with support contained in an interval  $[0, M]$ , and a signal of value  $v$ , drawn from a distribution with support contained in an interval  $[0, m]$ . If there is uncertainty on the costs or the outcome of an

investment, these values can be interpreted as expected values. The firm observes  $(c, v)$ , but the patent authority does not. We scale  $v$  to be the per-period monopoly profit that a patent would earn. We call  $(c, v)$  an idea.

The patent authority cannot observe  $(c, v)$  either *ex ante* or *ex post*, and the firm does not face the threat of entry either *ex ante* or *ex post*. The patent authority cannot base transfers on *ex post* information. If the patent authority could observe the R&D cost *ex post* (where “R&D cost” means the minimum reimbursement that the firm would require to undertake the project), the optimal policy would be to reimburse that cost, using optimal taxation. If the patent authority could observe the social value *ex post*, and if optimal taxation were costless, then the optimal policy would be to pay the social value.<sup>2</sup>

A *decision rule* is  $d_\delta: [0, M] \times [0, m] \rightarrow \{1, 0\}$ ,  $\delta: [0, m] \rightarrow \mathbb{R}_+$ , where  $d_\delta(\cdot) = 1$  means that the firm should invest and

$$\begin{aligned} d_\delta(c, v) &= 1 && \text{if } c \leq \delta(v) \\ d_\delta(c, v) &= 0 && \text{if } c > \delta(v). \end{aligned} \tag{1}$$

I thus restrict attention to decision rules that assign probability 1 or 0 to each reported type  $(\tilde{c}, \tilde{v})$ . The value  $\delta(v)$  represents the maximum R&D cost that the mechanism will support for an innovation of value  $v$ . If  $\delta(0) > 0$ , then firms would be subsidized to invest in innovations with value  $v = 0$ . Such subsidies could be exploited by opportunistic firms with no hope of achieving valuable innovations. For some of the discussion below, I assume that  $\delta(0) = 0$ .

A *mechanism* is  $(x, T, d_\delta)$ ,  $d_\delta$  as above,

$$x: [0, M] \times [0, m] \rightarrow \mathbb{R}, \quad T: [0, M] \times [0, m] \rightarrow \mathbb{R}_+,$$

where  $x(\cdot)$  is a transfer from the mechanism designer to the firm, and  $T(c, v)$  is the (discounted) length of a patent, namely

$$T(c, v) = \frac{1}{r} [1 - e^{-r\hat{T}(c,v)}]$$

when  $\hat{T}(c, v)$  is the undiscounted length of time. (But I use  $T$  instead of  $\hat{T}$ .)

I will use “ $\sim$ ” over an argument to represent a reported value rather than a true value. The firm’s profit is represented by the function  $\tilde{\Pi}: [0, M] \times [0, m] \times [0, M] \times [0, m] \rightarrow \mathbb{R}$ , where the first two arguments are reported values and the second two are true values. Thus

$$\tilde{\Pi}(\tilde{c}, \tilde{v}; c, v) \equiv x(\tilde{c}, \tilde{v}) + d_\delta(\tilde{c}, \tilde{v}) [vT(\tilde{c}, \tilde{v}) - c].$$

I will write  $\Pi(c, v)$  for  $\tilde{\Pi}(c, v; c, v)$ , namely, the profit when the firm reports its type honestly. I say that the mechanism  $(x, T, d_\delta)$  is *individually rational* if

<sup>2</sup> This is the basis of a scheme proposed by Kremer (1998), where transfers themselves are based on *ex post* information about  $v$ . Therefore a short period of profit can be inflated to give a reward based on  $v$  that bypasses the deadweight loss of a patent.

$$\Pi(c, v) \geq 0 \quad \text{for all } (c, v) \in [0, M] \times [0, m]$$

and is *incentive compatible* if

$$\Pi(c, v) \geq \tilde{\Pi}(\tilde{c}, \tilde{v}; c, v) \quad \text{for all } (\tilde{c}, \tilde{v}), (c, v) \in [0, M] \times [0, m].$$

Our first observation is that the mechanism must typically use patents ( $T(\cdot) > 0$ ). The intuition behind the following proposition is straightforward. The hypothesis is that we want to give higher compensation to innovators with higher-value innovations ( $\delta$  is increasing at some  $v$ ). This is because we are willing to tolerate higher R&D costs (as high as  $\delta(v)$ ) for higher-value innovations. But the firm's compensation can only depend on the true  $v$  if the firm can be punished for lying about  $v$ . Patents provide such punishment: if the firm reports  $\tilde{v} > v$  in order to get more compensation, its larger compensation will come partly in the form of a patent, whose value is less if the true value is  $v$  rather than  $\tilde{v}$ .

*Proposition 1.* Consider a decision rule  $d_\delta$  such that there exists a neighborhood in  $[0, m]$  where  $\delta$  is increasing. Then there is no incentive-compatible and individually rational mechanism  $(x, T, d_\delta)$  such that  $T(\cdot) \equiv 0$ .

*Proof.* See the Appendix.

### 3. Direct mechanisms

■ Using the revelation principle, I characterize the set of feasible decision rules by restricting attention to mechanisms that are incentive compatible. I show that the only decision rules that can be implemented are those with convex  $\delta$  and that the patent life in the corresponding mechanism is the derivative of  $\delta$ , which is nondecreasing. Higher-value innovations have longer patent lives. If  $\delta$  is not differentiable at  $v$ , its left and right derivatives nevertheless exist because  $\delta$  is convex. The patent life can be either the left or right derivative. The right derivative is defined by  $\delta'_R(v) = \lim_{v^n \rightarrow v} \delta'(v^n)$ , where  $v^n > v$ , and similarly for the left derivative. I define  $\delta'_L(0) = 0$ .

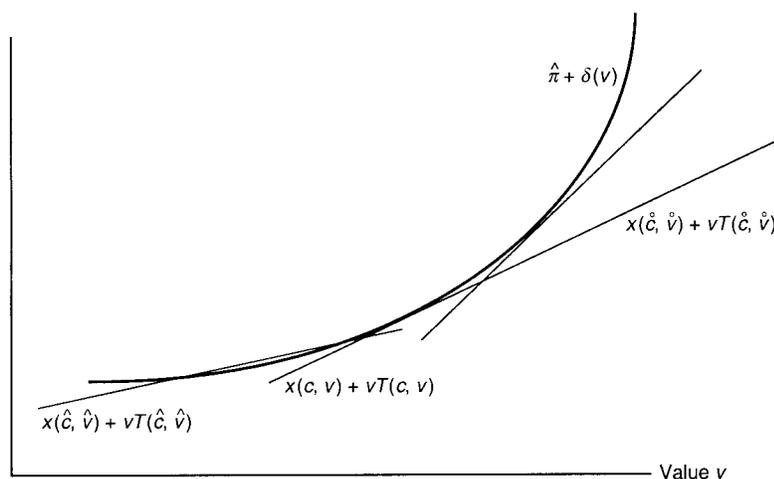
*Proposition 2.* Suppose that the mechanism  $(x, T, d_\delta)$  is incentive compatible and individually rational. Then

- (i)  $\Pi(c, v) = \max\{\hat{\pi}, \hat{\pi} + \delta(v) - c\}$  for some  $\hat{\pi} \geq 0$ .
- (ii)  $\delta$  is nondecreasing and convex.
- (iii)  $T(c, v) = \delta'(v)$  if  $\delta$  is differentiable at  $v$ . If  $\delta$  has a kink at  $v$ , then either  $T(c, v) = \delta'_R(v)$  or  $T(c, v) = \delta'_L(v)$ .

The proposition is proved in the Appendix, but perhaps the following intuition will suffice.

Figure 1 shows a convex function  $\delta$ , drawn as an upper envelope of linear functions. Each reported value  $(\tilde{c}, \tilde{v})$  defines a constant term, which is the transfer  $x(\tilde{c}, \tilde{v})$ , and slope, which is the patent life  $T(\tilde{c}, \tilde{v})$ , for a linear function  $v \mapsto x(\tilde{c}, \tilde{v}) + vT(\tilde{c}, \tilde{v})$ . For a fixed report  $(\tilde{c}, \tilde{v})$ , each linear function describes the firm's revenue, as it depends on the true value  $v$ . The firm's revenue rises along the linear function because the patent becomes more valuable with the true value  $v$ . A point on the envelope  $\delta$  is a point on one of the linear revenue functions, namely, the linear function (hence the report  $(\tilde{c}, \tilde{v})$ ) that maximizes revenue. Incentive compatibility means that at each  $v$ , the linear function with maximal value (which is a point on  $\delta$ ) is defined by the true values  $(c, v)$ . The firm's revenue is  $\delta(v) = \max_{\tilde{v}} x(\tilde{c}, \tilde{v}) + vT(\tilde{c}, \tilde{v})$ , which does not depend

FIGURE 1

INCENTIVE COMPATIBILITY AND CONVEX  $\delta$ 

on the true cost  $c$ . Hence, any cost savings due to having an “efficient” (low-cost) project will accrue to the firm.

In part (i),  $\hat{\pi}$  is the firm’s transfer (profit) if it reports values  $(\check{c}, \check{v})$  that do not result in investment, that is,  $d_\delta(\check{c}, \check{v}) = 0$ . Since the mechanism implements  $d_\delta$ , the maximum supported cost is  $c = \delta(v)$ . The firm’s profit at  $c = \delta(v)$  must be  $\hat{\pi}$ . If the profit were lower, the firm would overstate cost in order to avoid investing. If higher, a firm would report  $\check{c} = \delta(v)$ , even if  $\check{c}$  were slightly greater than  $\delta(v)$ , and then the mechanism would not implement  $d_\delta$ . Part (i) follows from two observations, first that a firm with an idea  $(\delta(v), v)$  must earn profit  $\hat{\pi}$ , and second that any efficiency surplus  $\delta(v) - c$  accrues to the firm because the revenue cannot depend on the true cost  $c$ .

In part (ii), the envelope  $\delta$  is nondecreasing because each linear function is nondecreasing, and convex because  $\delta$  is the upper envelope of linear functions.<sup>3</sup>

Figure 1 also shows part (iii), that the slope of  $\delta$  is the patent life. At each  $v$  the slope of the upper envelope is the slope of the linear function with the highest value at that  $v$ , namely  $T(\cdot, v)$ . If  $v$  is at a kink, then two linear functions have the same maximum value at that  $v$ , and the patent life can be taken as the slope of either of them.

There is no point giving more money to innovators than necessary, so we will henceforth assume that an incentive-compatible mechanism  $(x, T, d_\delta)$  is characterized by Proposition 2 with  $\hat{\pi} = 0$ , that is, the profit satisfies  $\Pi(c, v) = \max\{0, \delta(v) - c\}$ .

There are two consequences of Proposition 2 (incentive compatibility) that lead to the conjecture that all implementable decision rules can be implemented with renewal mechanisms. First, higher-value innovations receive longer patents. This is an inevitable outcome of a renewal system. Second, since the firm’s total compensation must be  $\delta(v)$  (else the mechanism does not implement  $d_\delta$ ), and since the firm earns some of the compensation in the form of a patent of value  $v\delta'(v)$ , the transfer must be  $\delta(v) - v\delta'(v)$ , which is nonincreasing with  $v$ . The firm pays money back to the patent authority in

<sup>3</sup> Convexity of  $\delta$  uses linearity of the payoff function. Such convexity was used in my (1992) article to show a regressive bias in tax enforcement, and the same convexity appears in Rochet and Choné (1998) in the monopoly problem.

order to prolong the patent. If  $\delta(0) = 0$ , the transfers at each  $v$  are nonpositive, and if  $\delta(0) > 0$ , the initial transfer is a subsidy.

#### 4. Renewal mechanisms

■ I showed above that the only decision rules  $d_\delta$  implementable with revelation mechanisms are those with convex  $\delta$ . I now show how those decision rules are implemented in the “real world,” namely with renewal mechanisms. Revelation mechanisms describe decision rules that are implementable, and the propositions in this section say that all implementable decision rules can be implemented with renewal mechanisms. The two are thus equivalent.

A *renewal mechanism* is  $(\Gamma, t_f)$ , where  $\Gamma: [0, t_f] \rightarrow \mathbb{R}$  is a nondecreasing function and  $t_f \in \mathbb{R}_+$  is the maximum termination date.  $\Gamma(t)$  is the total (discounted) payment required to extend the patent to time  $t$ . The firm can continue its patent to any termination date  $t \leq t_f$  provided it pays a fee with present value  $\Gamma(t)$ . If  $\delta$  has kinks, then  $\Gamma$  will have lump-sum payments at certain dates. At points where  $\Gamma$  is not continuous, I assume that the points of closure are on the left. That is, if  $\Gamma$  has a discontinuity at  $t$ , and the firm terminates the patent at  $t$ , then the lump-sum payment is avoided. The lump sum is paid only if the patent is renewed past  $t$ .

We can say that the renewal mechanism  $(\Gamma, t_f)$  implements a decision rule  $d_\delta$  if for each  $v \in [0, m]$

$$\max_{t^* \leq t_f} \left[ \int_0^{t^*} v e^{-rt} dt - \Gamma(t^*) \right] = \delta(v). \quad (2)$$

If this condition holds, the firm will invest in the innovation and maintain the patent for the optimal length of time if and only if  $c \leq \delta(v)$ .

The following proposition defines the renewal fees  $\Gamma$  that implement a particular  $d_\delta$ . The definition of  $\Gamma$  is complicated because  $\delta$  might have kinks and linear segments. After presenting the proof, I exhibit an equivalent mechanism with flow payments instead of lump-sum payments.

*Proposition 3.* Suppose that  $\delta$  is convex. Then there is a renewal mechanism  $(\Gamma, t_f)$  that implements the decision rule  $d_\delta$  with the same patent lives and profits as an incentive-compatible and individually-rational  $(x, T, d_\delta)$ .

*Proof.* Recalling that  $v \in [0, m]$ , define  $t_f$  by

$$\delta'_L(m) = \frac{1}{r} [1 - e^{-rt_f}]. \quad (3)$$

Define functions  $t^*: [0, m] \rightarrow \mathbb{R}_+$  and  $v^*: \mathbb{R} \rightarrow [0, m]$  by

$$\delta'_L(v) = \frac{1}{r} [1 - e^{-rt^*(v)}] \quad (4)$$

and

$$v^*(t) = \inf \{ v \in [0, m] \mid t^*(v) \geq t \}. \quad (5)$$

Define the lump-sum fees  $\Gamma$  by

$$\Gamma(t) = v^*(t)\delta'_L(v^*(t)) - \delta(v^*(t)). \quad (6)$$

The functions  $v^*(\cdot)$  and  $t^*(\cdot)$  are nondecreasing, since  $\delta'_L(\cdot)$  is nondecreasing. By definition of  $v^*$ ,  $t^*(v^*(t)) \geq t$ .

The function  $t^*(\cdot)$  will give optimal termination dates. If a type- $v$  firm terminates at  $t^*(v) = t$ , then the fee  $\Gamma(t)$  is  $v\delta'_L(v) - \delta(v)$ . However, the fees  $\Gamma$  must be defined for every  $t$ , including dates where no firm terminates, that is, dates  $t$  such that  $t \neq t^*(v)$  for all  $v$ . If no firm terminates at  $t$ , then  $v^*(t)$  is the smallest  $v$  that terminates later, and  $\Gamma(t)$  is defined with reference to  $v^*(t)$ .

We must verify two things: that (2) holds and that  $t^*(v)$  is the optimal termination date for a type- $v$  firm. If so, patent lives are the same as in the direct mechanism that implements  $d_\delta$ . First I verify (2), assuming that  $t^*(v)$  is the optimal stopping time for  $v$ . By definition of  $t^*$ ,  $\int_0^{t^*(v)} ve^{-rt} dt = v(1/r)[1 - e^{-rt^*(v)}] = v\delta'_L(v)$ , and by definition of  $\Gamma$  (using the fact that  $t^*$  is nondecreasing),  $\Gamma(t^*(v)) = v\delta'_L(v) - \delta(v)$ , hence (2) holds if  $t^*(v)$  is optimal.

Now I show that  $t^*(v)$  is in fact an optimal stopping time for  $v$ . Since a type- $v$  innovator can earn net revenue  $\delta(v)$  by stopping at  $t^*(v)$ , it is enough to show that no other stopping time provides greater net revenue than  $\delta(v)$ . That is, I must show that for all  $t$ ,

$$v\frac{1}{r}[1 - e^{-rt}] - \Gamma(t) = v\frac{1}{r}[1 - e^{-rt}] - v^*(t)\delta'_L(v^*(t)) + \delta(v^*(t)) \leq \delta(v). \quad (7)$$

I consider two cases, (i)  $t^*(v) > t$  and (ii)  $t^*(v) < t$ .

*Case (i).* Since  $t^*(v) > t$ , it follows from the definition of  $v^*$  that  $v \geq v^*(t)$ , so by monotonicity of  $t^*$ ,  $t^*(v) \geq t^*(v^*(t)) \geq t$ . Using convexity of  $\delta$ , the following holds, which implies (7).

$$\begin{aligned} v\frac{1}{r}[1 - e^{-rt}] - v^*(t)\delta'_L(v^*(t)) &\leq v\frac{1}{r}[1 - e^{-rt^*(v^*(t))}] - v^*(t)\delta'_L(v^*(t)) \\ &= [v - v^*(t)]\delta'_L(v^*(t)) \leq \delta(v) - \delta(v^*(t)). \end{aligned}$$

*Case (ii).* Since  $t^*(v) < t \leq t^*(v^*(t))$ , it follows by monotonicity of  $t^*$  that  $v \leq v^*(t)$ . Using convexity of  $\delta$ , the following holds, which implies (7).

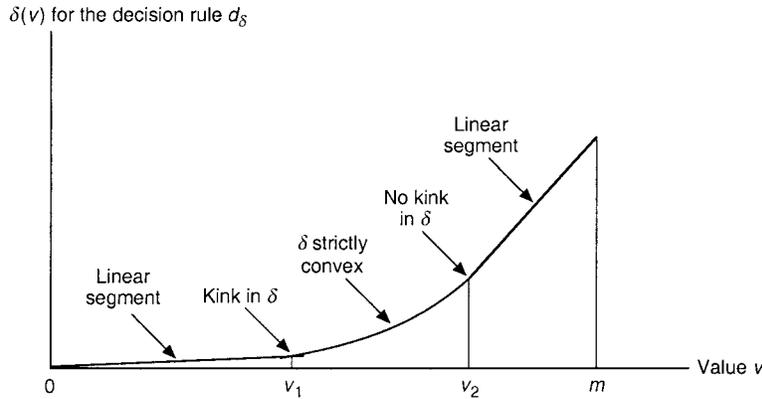
$$\delta(v^*(t)) - \delta(v) \leq \delta'(v^*(t))[v^*(t) - v] \leq \delta'_L(v^*(t))v^*(t) - v\frac{1}{r}[1 - e^{-rt}].$$

*Q.E.D.*

The proposition is illustrated in Figures 2 and 3. If  $\delta$  is linear on an interval, then all firms in the interval will terminate at the same time (the first and last segments in Figure 3). If  $\delta$  has a kink ( $v_1$  in Figure 2), then there is an interval of time where no firm terminates.

The lump-sum fees  $\Gamma$  are also equivalent to flow fees  $\gamma$ , which are nonnegative if  $\delta(0) = 0$ . If the derivative of  $\Gamma$  exists, then the flow of (discounted) fees at time  $t$  is  $\Gamma'(t)$ . But one of the difficulties in defining the renewal fees as a flow is that the derivative might not exist, in particular because  $\Gamma$  takes jumps if  $\delta$  has kinks. Nevertheless there is a renewal mechanism with flow payments that is equivalent to  $(\Gamma, t_f)$ . The flow payments  $\gamma$  will be nondecreasing.

FIGURE 2  
CUTOFF  $\delta$  FOR THE DECISION RULE  $d_\delta$



I define a *renewal mechanism with flow payments*  $(\gamma, t_f)$  as  $\gamma: [0, t_f] \rightarrow \mathbb{R}_+$  and  $t_f \in \mathbb{R}_+$ , where  $\gamma(t)$  is the flow of fees paid at time  $t$ , and all flow payments until  $t$  must be paid in order to renew the patent until  $t$ . We can say that the renewal mechanism  $(\gamma, t_f)$  implements  $d_\delta$  if for each  $v \in [0, m]$ ,

$$\max_{t^* \leq t_f} \left[ \int_0^{t^*} [v - \gamma(t)]e^{-rt} dt \right] = \delta(v). \tag{8}$$

The argument behind the following remark can be found in Scotchmer (1997). There are some technical matters relating to kink points in  $\delta$ , but other than that,  $\gamma$  is chosen so that it integrates to  $\Gamma(t)$  at each  $t$ .

*Remark 1.* Suppose that  $\delta$  is convex and  $\delta(0) = 0$ . The following renewal mechanism with flow payments,  $(\gamma, t_f)$ , implements  $d_\delta$  with the same patent lives and total profits as the incentive-compatible, individually-rational direct mechanism  $(x, T, d_\delta)$  and the equivalent renewal mechanism  $(\Gamma, t_f)$ .

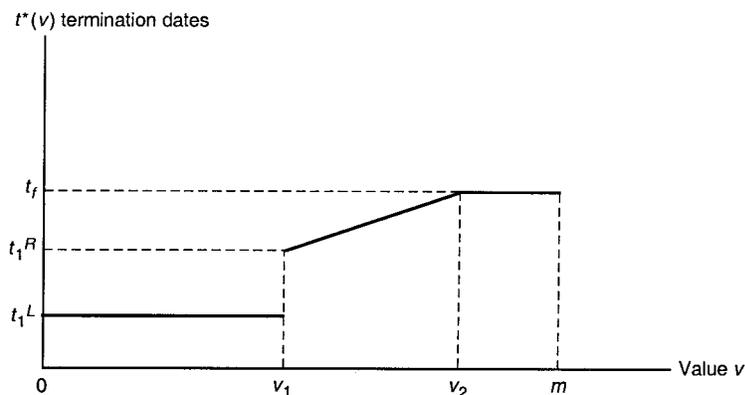
$$\gamma(t) = v^*(t), \tag{9}$$

where  $v^*(\cdot)$  is defined in (5) and  $t_f$  is defined in (3).

### 5. The optimal incentive system

■ In Proposition 2 I showed that in an incentive-compatible mechanism, the decision rule  $d_\delta$  has convex  $\delta$ . But which  $\delta$  is optimal? In this section I investigate the efficient decision rule in two research environments, one in which the costs and benefits are independently distributed, and another in which costs are a deterministic function of the value. In the first case, it will be optimal to present a trivial renewal system where only one choice is offered, that is, the patent life is fixed. In the second case, it will be optimal to present a nontrivial menu of choices, as is implicit in a renewal system. If subsidies are feasible, low-value innovators will receive pure subsidies and high-value innovators will choose patents with smaller (possibly negative) subsidies. It might seem counterintuitive that low-value innovations should be subsidized, but a benefit

FIGURE 3

OPTIMAL TERMINATION DATES  $t^*(\cdot)$ 

arises from the incentive constraint: with subsidies to low-value innovators, the fees paid by high-value innovators can be lower, and patents on high-value innovations can be shorter.

To investigate the efficient decision rule, we must know how innovations contribute to social welfare. I have defined  $v$  to be the per-period monopoly profit available from a patent on the innovation. The per-period social value includes consumer surplus, and it will be larger than  $v$  when the new product is sold by a monopolist, assuming the monopolist cannot perfectly price discriminate. The social value will be even larger in a competitive market, since the deadweight loss of monopoly pricing is then avoided. I assume that there are constants  $k_2 > 0$ ,  $k_1 > 1$ , such that the per-period social value is  $(k_1 - k_2)v > v$  in periods when the product is sold by a monopolist and is  $k_1v$  if sold by competitive firms. Over the (infinite) lifetime of the product, the social value is therefore

$$\text{social value} = \left( \frac{k_1}{r} - k_2 T(c, v) \right) v - c,$$

where  $c$  is the R&D cost and  $T(c, v)$  is the discounted life of the patent.

The social value assumes that pure transfers from taxpayers to innovators, subsidies, are not costly from a social point of view. The only costs are deadweight loss from monopoly pricing and resources spent on R&D. Such an objective function should bias the optimal policy in favor of subsidies rather than patents. Interestingly, the next section shows that even with this bias, subsidies (if allowed) will be supplemented by patents. Patents cause firms to self-select investment in a way that compares the value of the innovation to its R&D cost. The problem with subsidies is that they support investment even if the innovation's value does not justify the R&D cost. Subsidies cannot favor ideas that, although they have high cost, also have high value.

A firm's total compensation (fees plus patent) is  $\delta(v)$  and there are research subsidies if  $\delta(0) > 0$ . Opportunistic firms with zero-value ideas receive  $\delta(0)$ , which they keep even if they produce nothing of value. The problem with subsidies, and an argument for restricting  $\delta(0) = 0$ , is that subsidies could make the patent authority hemorrhage money. Unless the patent authority can prescreen the firms in some way that is equivalent to observing  $(c, v)$  before offering the mechanism, an unlimited

number of firms might show up to claim the subsidy and not produce worthwhile innovations.

□ **Independently distributed costs and benefits.** Suppose now that ideas  $(c, v)$  for R&D projects occur randomly, and that  $(c, v)$  are independently and uniformly distributed on the supports  $[0, M]$  and  $[0, m]$  (ignoring the opportunistic firms clustered at  $(0, 0)$ ). The social welfare of the decision rule  $d_\delta$  is

$$\begin{aligned} W(\delta) &= \int_0^m \int_0^{\delta(v)} \left\{ \left[ \frac{k_1}{r} - k_2 T(c, v) \right] v - c \right\} \frac{1}{M} dc \frac{1}{m} dv \\ &= \frac{1}{mM} \int_0^m \left\{ \delta(v) \left[ \frac{k_1}{r} - k_2 \delta'(v) \right] v - \frac{1}{2} \delta(v)^2 \right\} dv. \end{aligned} \quad (10)$$

We seek the convex, nondecreasing function  $\delta$  that maximizes (10). It turns out to be linear, i.e., it has a fixed patent life. In the following propositions I assume that

$$k_1/[r(1 + 2k_2)]m \leq M$$

and

$$[2(k_1/r)k_2m]/[(1 + 3k_2)(1 - k_2)] + [(k_1/r)(1 - 3k_2)]/[(1 + 3k_2)(1 - k_2)]m \leq M,$$

so there is no problem of bumping up against boundaries.

*Proposition 4 (with uniformly distributed costs and benefits, a uniform patent life is optimal).* Suppose that  $(c, v)$  are independently and uniformly distributed on the supports  $[0, M]$  and  $[0, m]$ . In the class of piecewise twice-differentiable functions  $\delta: [0, m] \rightarrow \mathbb{R}_+$  such that  $\delta(0) = 0$  and  $\delta$  is nondecreasing and convex, the one that maximizes social welfare (10) satisfies

$$\delta(v) = \frac{k_1}{r(1 + 2k_2)}v. \quad (11)$$

*Proof.* See the Appendix.

For the case that subsidies are allowed ( $\delta(0) > 0$ ), the following proposition says that it is better to give some of the reward as a subsidy and some of the reward as a shorter patent. (The patent life in (12) is smaller than in (11).) If monopoly distortions are costly enough, it is optimal to not give a patent at all, but only a subsidy. The following is proved using the same techniques as Proposition 4, without the restriction  $\delta(0) = 0$ .

*Proposition 5.* Suppose that  $(c, v)$  are independently and uniformly distributed on the supports  $[0, M]$  and  $[0, m]$ . In the class of piecewise twice-differentiable functions  $\delta: [0, m] \rightarrow \mathbb{R}_+$  such that  $\delta(0) \geq 0$  and  $\delta$  is nondecreasing and convex, the one that maximizes social welfare (10) satisfies

$$\delta(v) = \frac{2(k_1/r)k_2m}{(1 + 3k_2)(1 - k_2)} + \frac{(k_1/r)(1 - 3k_2)}{(1 + 3k_2)(1 - k_2)}v \quad \text{if } k_2 \leq 1/3 \quad (12)$$

$$\delta(v) = \frac{k_1}{r} \frac{m}{2} \quad \text{if } k_2 \geq 1/3. \quad (13)$$

□ **Efficient decision rule with deterministic cost functions.** I now consider two cases with deterministic cost functions  $g$ , where  $g(v)$  is the cost (formerly  $c$ ) of an innovation with value  $v$ . The two cases are that  $g$  is concave and  $g$  is convex. Social welfare is then the following, where  $v$  is distributed according to the distribution  $H$ , and  $d_\delta(g(v), v) = 1$  if and only if  $\delta(v) \geq g(v)$ . The welfare function reflects the fact that the patent life is  $\delta'(v)$ .

$$W(\delta) = \int_0^m d_\delta(g(v), v) \left\{ \left[ \frac{k_1}{r} - k_2 \delta'(v) \right] v - g(v) \right\} dH(v). \quad (14)$$

First suppose that  $g$  is concave, as in Figure 4, so that the cost of innovations increases less than proportionately with their value. For example, it might be that high-value innovations are largely the result of serendipity rather than hard work, whereas low-value innovations result from investments building on previous knowledge.

With concave  $g$  it is impossible to implement the decision rule  $d_g$ . For  $\delta(0) = 0$  (no subsidy), the optimal mechanism has a fixed patent life. This is shown by the dashed line in Figure 4, where the slope of the line is the patent life. Only investments in high-value innovations are supported. There is clearly no advantage in allowing  $\delta$  to be strictly convex. The linear function  $\delta$  has the minimum slope (patent life) among all functions  $\delta$  such that  $\delta(0) = 0$  and  $\delta(v) > g(v)$  for  $v \geq \bar{v}$ .

If the patent authority allows  $\delta(0) > 0$  (subsidy), then the patent life for the supported high-value ideas  $v \geq \bar{v}$  can be shortened: The slope of the dotted line  $\hat{\delta}$  for  $v \geq \bar{v}$  is smaller than the slope of the dashed line. However, some low-value innovations ( $v < \bar{v}$ ) are subsidized despite their low value. The renewal-system interpretation of  $\hat{\delta}$  in Figure 4 is that all innovators will receive the subsidy, but high-value innovators will pay something back to the patent authority in return for a patent. Low-value innovators choose the subsidy but no patent.

FIGURE 4

CONCAVE COST FUNCTION  $g$

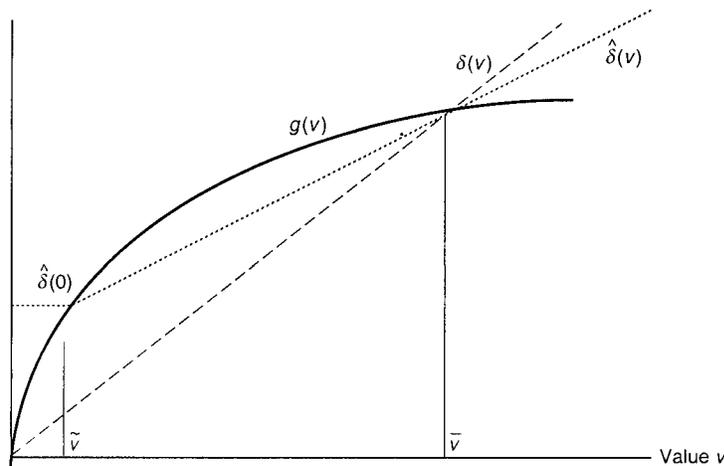
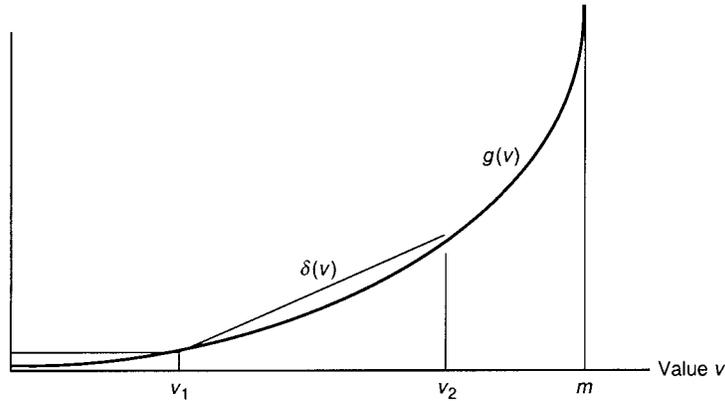


FIGURE 5  
CONVEX COST FUNCTION  $g$



*Remark 2.* Suppose that the R&D cost function  $g$  is concave. With no subsidies allowed, the optimal renewal system is a fixed patent life. If subsidies are allowed, the optimal renewal system is a menu with two options: a large subsidy with no patent or a patent plus a smaller subsidy (or fee). Low-value innovations will choose only the subsidy, while higher-value innovations will choose a lower (possibly negative) subsidy with a patent.

Now suppose that  $g$  is convex, as in Figure 5, so that the cost of innovations increases more than proportionately with the value. Since  $g$  is convex,  $d_g$  is implementable. That is, it is possible to elicit investment in every innovation  $v$  while reimbursing only the costs  $g(v)$ , and not more. Although such a mechanism might seem optimal, Figure 5 shows two reasons why it is not.

First, if subsidies are allowed, it is optimal to use them, provided  $g'(v)$  is small in a neighborhood of  $v = 0$  and  $g(0) = 0$ . Under these conditions, there will be some domain  $(0, v_1)$  where it is optimal to support investment, and the efficient way to support investment at low  $v$  is to provide subsidies. Subsidies avoid deadweight loss on low-value innovations, without constraining  $\delta$  at  $v > v_1$ . This is shown in Figure 5. Second, provided the function  $v \mapsto vg'(v)$  is also convex and  $v$  is uniformly distributed, the optimal renewal scheme will have discrete jumps in patent lives instead of differentiating patent lives smoothly according to  $v$ . Consider  $v \in (v_1, v_2)$  in Figure 5. If the patent authority substitutes a linear function  $\delta$  for the convex function  $g$ , as shown, then it pays more money to the firm ( $g(v) < \delta(v)$ ) but reduces average patent lives (this follows from Jensen's inequality). We thus have the following:

*Remark 3.* Suppose that R&D costs are given by a strictly convex and smooth function  $g$  such that  $g(0) = 0$ ,  $g'(0) = 0$ , and  $v \mapsto vg'(v)$  is convex. Suppose that  $v$  is uniformly distributed. Then the optimal renewal mechanism is a menu with discrete jumps in patent lives. At zero patent life there is a subsidy. Firms can choose higher patent lives in return for lower (possibly negative) subsidies.

**Appendix**

■ Proofs of Propositions 1, 2, and 4 follow.

*Proof of Proposition 1.* Let

$$\begin{aligned}
 I &\equiv \{(\bar{c}, \bar{v}) \in [0, M] \times [0, m] \mid d_\delta(\bar{c}, \bar{v}) = 1\} \\
 N &\equiv \{(\bar{c}, \bar{v}) \in [0, M] \times [0, m] \mid d_\delta(\bar{c}, \bar{v}) = 0\}.
 \end{aligned}
 \tag{A1}$$

The profit function without patents is  $\tilde{\Pi}(\tilde{c}, \tilde{v}; c, v) = x(\tilde{c}, \tilde{v}) - d_s(\tilde{c}, \tilde{v})c$ . If the firm reports  $(\tilde{c}, \tilde{v}) \in I$ , then it will be told to invest, and must bear cost  $c$ , and if it reports  $(\tilde{c}, \tilde{v}) \in N$ , it will be told not to invest, and avoids cost  $c$ . Conditional on reporting any  $(\tilde{c}, \tilde{v}) \in I$  (respectively in  $N$ ), the firm will report the  $(\tilde{c}, \tilde{v}) \in I$  that maximizes the transfer. Thus the transfer  $x(\tilde{c}, \tilde{v})$  must be constant on  $I$  and  $N$  respectively. Call these transfers respectively  $x^I$  and  $x^N$ . Suppose that  $\hat{v}$  is in the neighborhood where  $\delta$  is increasing. Then there exist  $(c, v) \in N$ ,  $(c^*, v^*) \in I$  such that  $c < c^*$ ,  $v < v^*$ . But by incentive compatibility,  $x^I - c \leq x^N$ , and  $x^I - c^* \geq x^N$ , hence  $c \geq c^*$ , a contradiction. *Q.E.D.*

*Proof of Proposition 2.* (i) First, there exists  $\hat{\pi} \geq 0$  such that  $\Pi(c, v) = \hat{\pi}$  for all  $(c, v) \in N$  defined in (A1). If profit were not the same for all points in  $N$ , then, conditional on reporting some point  $(\tilde{c}, \tilde{v}) \in N$ , the firm would report the most profitable point, irrespective of the true  $(c, v)$ .

Further, for each  $v \in [0, m]$ ,  $\Pi(\delta(v), v) = \hat{\pi}$ . If  $\Pi(\delta(v), v) < \hat{\pi}$ , then the firm of type  $(\delta(v), v)$  would report some  $(\tilde{c}, \tilde{v}) \in N$  rather than  $(\delta(v), v)$ . If  $\Pi(\delta(v), v) - \hat{\pi} = \epsilon > 0$ , then a firm of type  $(\delta - \epsilon/2, v)$  would report  $(\delta(v), v)$  instead of  $(\delta(v) - \epsilon/2, v)$ .

We now observe that for given  $v$  and  $c \leq \delta(v)$ ,  $\Pi(c, v) = k(v) - c$  for some real value  $k(v)$ . In an incentive-compatible mechanism,  $\tilde{v} \equiv v$ . Given  $v$ , the revenue  $x(\tilde{c}, v) + T(\tilde{c}, v)v$  must have the same value for all  $\tilde{c} \leq \delta(v)$ . If  $x(\tilde{c}, v) + T(\tilde{c}, v)v < x(\hat{c}, v) + T(\hat{c}, v)v$ , then the firm would report  $(\hat{c}, v)$  rather than  $(\tilde{c}, v)$ , even if the true  $c$  were  $\tilde{c}$ . Thus, for given  $v$  and  $c \leq \delta(v)$ ,  $\Pi(c, v) = k(v) - c$  for some real value  $k(v)$ . Finally, since  $\Pi(\delta(v), v) = \hat{\pi} = k(v) - \delta(v)$ , it must hold that  $k(v) = \hat{\pi} + \delta(v)$ .

(ii) I now show that  $\delta$  is nondecreasing and convex. Using part (i), if  $\delta$  were decreasing, then there exist  $v < v'$  and  $c < v$  such that  $\Pi(c, v) > \Pi(c, v')$ . But since  $T(\cdot) \geq 0$ ,

$$\Pi(c, v) = \tilde{\Pi}(c, v; c, v) = \tilde{\Pi}(c, v; c, v') \leq \Pi(c, v'),$$

this is a contradiction.

To show convexity, consider three points,  $v_1, v_c, v_2$ , where  $v_c = \alpha v_1 + (1 - \alpha)v_2$ . I use incentive compatibility and the fact that  $\Pi(\delta(v), v) = x(\delta(v), v) + vT(\delta(v), v) - \delta(v) = \hat{\pi}$ . Let

$$\delta_i = \delta(v_i), x_i = x(\delta_i, v_i), T_i = T(\delta_i, v_i) \quad \text{for } i = 1, c, 2.$$

Then

$$\begin{aligned} \delta_2 + \hat{\pi} &= x_2 + v_2 T_2 \geq x_c + v_2 T_c = \delta_c - (v_c - v_2)T_c + \hat{\pi} \\ \delta_1 + \hat{\pi} &= x_1 + v_1 T_1 \geq x_c + v_1 T_c = \delta_c - (v_c - v_1)T_c + \hat{\pi}. \end{aligned}$$

Multiplying the first equation by  $(1 - \alpha)$ , the second by  $\alpha$ , and adding,

$$(1 - \alpha)\delta_2 + \alpha\delta_1 + \hat{\pi} \geq \delta_c + \hat{\pi},$$

this shows that  $\delta$  is convex.

(iii)  $\tilde{\Pi}(\tilde{c}, \tilde{v}; c, v)|_{(\tilde{c}, \tilde{v})=(c, v)} = [x(\tilde{c}, \tilde{v}) + vT(\tilde{c}, \tilde{v}) - c]|_{(\tilde{c}, \tilde{v})=(c, v)} = \Pi(c, v) = \hat{\pi} + \delta(v) - c$ . In an incentive-compatible mechanism, as  $v$  changes, the reported value  $\tilde{v}$  will change one-to-one with  $v$  to maintain  $\tilde{v} \equiv v$ , and neither  $\tilde{c}$  nor  $c$  will change. Thus the partial derivative  $(\partial/\partial v)\tilde{\Pi}|_{(\tilde{c}, \tilde{v})=(c, v)}$  is  $T(\tilde{c}, \tilde{v}) = T(c, v)$ , which must equal the partial derivative  $(\partial/\partial v)\Pi(c, v) = \delta'(v)$  when  $\delta$  is differentiable.

If  $\delta$  has a kink at  $v$ , then one can make the above calculation for either the left or right derivative, and the mechanism could be defined such that the type- $(c, v)$  firm could choose the patent life that corresponds to either one. *Q.E.D.*

*Proof of Proposition 4.*<sup>4</sup> Introduce two control variables,  $u(\cdot)$  and  $w(\cdot)$ . Let

$$f(v, \delta, u) \equiv \delta \left[ \frac{k_1}{r} - k_2 u \right] v - \frac{1}{2} \delta^2.$$

The control variable  $u$  is equal to  $\delta'$ , and the control variable  $w$  is equal to  $u'$ . I use the control variable  $w$  to constrain  $u'$  (hence  $\delta''$ ) to be nonnegative. I let  $\lambda(\cdot)$  and  $\gamma(\cdot)$  be the costate variables associated respectively with the constraints  $\delta'(v) = u(v)$  and  $w(v) = u'(v)$ . I let  $\eta(\cdot) \geq 0$  be the shadow price on the inequality

<sup>4</sup> The maximum of (10) is not described by the Euler equation, as the solution does not satisfy the relevant endpoint condition at  $v = m$ . This is why I introduce constraints and costate variables.

constraint  $w(v) \geq 0$ . The solution will satisfy  $\delta(\cdot) > 0$  and  $u'(\cdot) \geq 0$ , so I will ignore these constraints in the optimization. The constraints are

$$\begin{aligned}\delta'(v) &= u(v) \\ u'(v) &= w(v) \\ u(v) &\geq 0 \\ w(v) &\geq 0 \\ \delta(v) &\geq 0 \\ \delta(0) &= 0.\end{aligned}$$

Following, for example, Kamien and Schwartz (1981), one can write the Hamiltonian

$$H(v, \delta(v), u(v), w(v), \lambda(v), \gamma(v)) = f(v, \delta(v), u(v)) + \lambda(v)u(v) + \gamma(v)w(v)$$

and form a Lagrangian as

$$L(v, \delta(v), u(v), w(v), \lambda(v), \gamma(v)) = f(v, \delta(v), u(v)) + \lambda(v)u(v) + \gamma(v)w(v) + \eta(v)w(v).$$

In addition to  $\delta'(v) = u(v)$ ,  $u'(v) = w(v)$ , the necessary conditions for a maximum are

$$\begin{aligned}\frac{\partial H}{\partial \delta} = f_\delta &= \left[ \frac{k_1}{r} - k_2 u(v) \right] v - \delta = -\lambda'(v) \\ \frac{\partial H}{\partial u} = f_u + \lambda &= -\delta k_2 v + \lambda = -\gamma'(v) \\ \frac{\partial H}{\partial w} = \gamma + \eta &= 0 \\ \eta(\cdot) &\geq 0.\end{aligned}$$

In addition, the costate variables  $\lambda$  and  $\gamma$  must have value zero at the endpoints where the associated-state variables  $\delta$  and  $u$  are unconstrained, thus

$$\lambda(m) = \gamma(0) = \gamma(m) = 0.$$

These necessary conditions are satisfied by  $\delta$  stated in the proposition and

$$\begin{aligned}\gamma(v) &= \frac{k_1 k_2 v}{2r(1 + 2k_2)}(v^2 - m^2) \\ \lambda(v) &= \frac{k_1 k_2}{2r(1 + 2k_2)}(m^2 - v^2) \\ \eta(v) &= -\gamma(v) \geq 0.\end{aligned}$$

Since the Hamiltonian is concave in  $u$  and  $w$ , these describe maxima. *Q.E.D.*

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