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## Patents or prizes: Monopolistic R&D and asymmetric information

Eric A.A. de Laat

*Tinbergen Institute, Erasmus University Rotterdam, P.O. Box 1738, 3000 DR Rotterdam,  
The Netherlands*

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### Abstract

In this paper a model of R&D regulation is analysed to compare the effect of two types of asymmetric information on the welfare properties of patents and prizes as R&D incentive instruments when there is a technological leader. Using the case of full information as a benchmark, it is found that the trade-off between patents and prizes does not change if the innovator's R&D costs are private information, whereas the relative efficiency of patents decreases if the government is less informed about the market for the innovation.

*Keywords:* Patents; Research prizes; Asymmetric information; R&D regulation

*JEL classification:* O31; O34; L51

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### 1. Introduction

The patent system is a widely used means of providing incentives for research and a large part of the literature concerned with stimulation of R&D—initiated by Nordhaus (1969)—focuses on patents.<sup>1</sup> But in view of the

<sup>1</sup> Providing R&D incentives is not the only function of patents. Scotchmer and Green (1990) discuss the disclosure of new knowledge that would otherwise have been kept secret; Kitch (1977) points at the “prospect role”: allowing an innovator to explore the development possibilities of a not yet marketable innovation; David and Olsen (1992) investigate whether a patent-based monopoly leads to faster learning, mitigating the welfare losses associated with patents. These functions of patents play no role in this paper.

considerable deadweight losses due to the monopoly power associated with the granting of a patent, it seems much cheaper in terms of welfare losses to let an innovation immediately become public property and to reward the innovator with a sum of money, i.e. to use a prize system.<sup>2</sup>

Is there an economic rationale for the use of patents instead of other instruments? The information problems that are typically associated with the process of innovation might provide this rationale.<sup>3</sup> Patents may be better suited to overcome these problems than other R&D policy instruments. Wright (1983) arrives at precisely this conclusion analysing a model in which he assumes perfect competition and free entry in the R&D process. He finds that the effect of asymmetric information between the government and the potential innovators depends on the nature of the private information. If the government is less informed about the costs to innovation, prizes or contracts are always better than patents: the instruments can generate the same reward structure, but the patent does so at a considerable welfare loss. If the information asymmetry pertains to the private and social benefits of the innovation, patents may be better: the value of a patent is linked to the benefit of the innovation, whereas the values of a prize and a contract are constant. This superior reward structure may offset the welfare losses.

A natural question is whether these results hold under less competitive R&D processes. Whereas Wright considers a competitive R&D process, this paper compares the effect of the two types of asymmetric information on the performance of patents and prizes in the case of one innovator, a *technological leader*. Contracts are not considered, since in this framework they are equivalent to prizes. R&D subsidies are also not considered, since the recipient will have no incentive to spend it on the R&D project, unless his spending behaviour is verifiable; in this latter case a subsidy is equivalent to a prize. A negative prize (profit tax) is not ruled out: the cases of non-negative and unrestricted prizes are both discussed.<sup>4</sup>

The monopolistic nature of the R&D process allows the problem of finding the optimal prizes and patent systems to be modelled as a regulation problem for the government. It commits to a prize and patent system, in reaction to which the innovator chooses the innovation it will pursue. The

<sup>2</sup> The amount of literature on research prizes is small compared to the patent literature; an example is Dixit (1988) who introduces awards as policy instruments to relieve the common pool problem in R&D races.

<sup>3</sup> Arrow (1962) was the first to point out the role of information asymmetries as a source of market failure. Since then several authors have indicated the importance of information for an efficient regulation of R&D, e.g. Sappington (1982), Wright (1983), Rafiquzzaman (1987), Scotchmer (1991) and Olsen (1993).

<sup>4</sup> Dixit (1988) also allows for negative awards. A possible objection to taxes as an instrument for innovation policy—pointed out to me by a referee—are the typically different time horizons for fiscal policy (short-term) and innovation policy (long-term).

innovation is modelled as a single variable, *innovation size*.<sup>5</sup> Since the innovator can choose between innovations, the government may want to condition the reward on the innovation. Thus it announces patent durations and prizes that are contingent on the innovation size.<sup>6</sup> The welfare costs associated with prizes consist of the distortions created by the tax system needed to collect the award money; the welfare costs associated with the patent system are the deadweight monopoly losses incurred during the patent's lifetime. The government maximizes net welfare, the innovator maximizes net profits.

The solution to the problems in this paper rests on techniques from the mechanism design and regulation literature.<sup>7</sup> The first result is that asymmetric information about the innovation costs has no effect on the welfare ranking of patents and prizes: the information externality is the same for the two instruments, leaving the trade-off between them unchanged. The second result is that asymmetric information about the market for the innovation *worsens* the efficiency of patents as an incentive instrument: only the use of patents entails an information externality in this case, so that the trade-off changes in favour of prizes. The first result is in accordance with Wright's findings, the second is in sharp contrast to them. Thus, asymmetric information about the market can only be used as a justification for the use of patents when the R&D process is sufficiently competitive.

The paper is organized as follows. In Section 2 the model is presented and the full information case is analysed. Sections 3 and 4 discuss the optimal R&D policy under asymmetric information about the innovation costs and about the market conditions, respectively, and compare the outcomes with the full information case. Section 5 concludes.

<sup>5</sup> For process innovations the size is the cost reduction achieved. Product innovations can be naturally thought of as bundles of new product characteristics. A firm only considers the most profitable bundles given the innovation costs. The innovation size can then be thought of to represent the bundles on this "efficient frontier". The speed with which a given innovation is performed provides another example of its size.

<sup>6</sup> Sappington (1982) also allows for variable patent durations. Two concepts justify the notion of a duration function: the protectiveness of a patent and the renewal fees required to continue its validity. The profitability of a patent will generally decrease over time as the renewal fees increase, competitors discover minor improvements or get better at inventing around the patent. Hence, the de facto expiration date of a patent is largely determined by the levels of protection and profitability. A prize function seems less likely to be implementable in practice, although one could think of different money prizes for different classes of research achievements. I thank Paul Klemperer for pointing out to me the duration determining property of renewal fees.

<sup>7</sup> See Myerson (1979), Baron and Myerson (1982) and Laffont and Tirole (1993).

## 2. The model and the case of full information

An industry is considered in which one firm—the technological leader—is capable of performing an innovation of variable size  $I \geq 0$ , that has the potential to increase per period welfare by an amount  $W(I|\zeta) = \zeta \cdot w(I)$ , which is a strictly increasing and concave function of the innovation size:<sup>8</sup>

$$w(0) = 0; \quad w'(I) > 0, \quad w''(I) \leq 0.$$

The parameter  $\zeta$  is a measure of market size:  $\zeta \in [\underline{\zeta}, \bar{\zeta}] \subset (0, \infty)$ . The results obtained in this paper do not depend on the time path of the welfare flow; therefore it is taken to be time independent. The firm's costs of performing an innovation are a function  $C(I|\theta) = \theta \cdot c(I)$ , that is strictly increasing and convex in the size of the innovation:<sup>9</sup>

$$c(0) = 0; \quad c'(I) > 0, \quad c''(I) \geq 0.$$

The parameter  $\theta$  denotes the innovator's cost effectiveness:  $\theta \in [\underline{\theta}, \bar{\theta}] \subset (0, \infty)$ . Both  $w(\cdot)$  and  $c(\cdot)$  are assumed to be twice continuously differentiable on  $[0, \infty)$ . Very large innovations are assumed to be prohibitively costly:

$$\lim_{I \rightarrow \infty} \frac{w(I)}{c(I)} = 0.$$

The per period monopoly profits the firm makes, if it is awarded a patent, are assumed to equal  $\pi(I|\zeta) \equiv \alpha \cdot W(I|\zeta)$ ,  $\alpha \in (0, 1]$ , where  $1 - \alpha$  represents the fraction of potential welfare lost due to monopoly behaviour.<sup>10</sup>

In the initial situation all firms in the industry make zero profits. Unprotected innovations are imitated instantaneously and costlessly, returning the industry to a new zero profit situation. Hence, to induce innovation the firm must be allowed to earn back its innovation costs by ways of government intervention. The government can issue a prize  $P$ , or a patent with duration  $T$ , or both. Patent lives must be non-negative ( $T \geq 0$ ). Prizes are either restricted to be non-negative ( $P \geq 0$ : taxes as an innovation policy instrument are ruled out), or allowed to take on negative values as well;

<sup>8</sup> This is a general version of the "invention possibility function" of Nordhaus (1969). This welfare can be thought of as the (additional) surplus-area under a demand curve made available by the innovation.

<sup>9</sup> Baron and Myerson (1982) use a cost function that is bilinear in output,  $q$ , and  $\theta$ , and not continuous at  $q = 0$ . Allowing for fixed costs and/or variable costs independent of  $\theta$  here does not change the results.

<sup>10</sup> There exist simple models in which this loss is constant; e.g. if demand is linear a non-drastring process innovation implies  $\alpha = 2/3$ . Allowing for different levels of welfare loss across innovations cannot be expected to change the results significantly: instead of comparing the tax distortion loss to a constant monopoly deadweight loss, it is compared to an average deadweight loss over the relevant innovation sizes.

both cases are discussed. The government maximizes net total discounted welfare, the innovator maximizes net total discounted profits. The social and private discount rate is denoted by  $r > 0$ . To simplify the exposition the government is assumed to prefer the incentive system with the largest prize amount if it is indifferent (equal welfare levels).

If the government uses a mechanism that rewards an innovation  $I$  with a prize  $P$  and a patent of duration  $T$ , the firm's net profit from innovation consists of this prize plus the monopoly profit flow during the life of the patent minus the innovation costs:

$$\begin{aligned}\Pi(I, P, T|\theta, \zeta) &= P + \int_0^T \pi(I|\zeta)e^{-rt} dt - \theta c(I) \\ &= P + \tau\alpha \frac{\zeta w(I)}{r} - \theta c(I),\end{aligned}\quad (1)$$

with  $\tau = 1 - e^{-rT}$  (the  $T$ -representation and  $\tau$ -representation of patent duration will be used interchangeably throughout this paper).

During the life of the patent, households do not benefit from the innovation so that the consumer surplus is the discounted welfare flow from expiration of the patent onwards, reduced by the taxes levied to raise the prize money:

$$\begin{aligned}CS(I, P, T|\zeta) &= \int_T^\infty W(I|\zeta)e^{-rt} dt - (1 + \lambda)P \\ &= (1 - \tau) \frac{\zeta w(I)}{r} - (1 + \lambda)P.\end{aligned}\quad (2)$$

The parameter  $\lambda \geq 0$  is the shadow cost of public funds, representing the social costs to taxation.<sup>11</sup>

Total social welfare is the sum of producer surplus (1) and consumer surplus (2):

$$\Omega(I, P, T|\theta, \zeta) = \frac{\zeta w(I)}{r} - \lambda P - \tau(1 - \alpha) \frac{\zeta w(I)}{r} - \theta c(I). \quad (3)$$

This expression illustrates the social costs associated with the two instruments: for prizes they are the distortionary taxation costs  $\lambda P$  and for patents the deadweight losses during the patent's life  $\tau(1 - \alpha)(\zeta w(I))/r$ .

To find the optimal full information (FI) prize and patent system for given

<sup>11</sup> Although Wright (1983) assumes this shadow cost to be equal to zero, Laffont and Tirole (1993, p. 38) mention studies that report that  $\lambda = 0.3$  seems to be a reasonable estimate for the U.S. economy. For a discussion of this shadow cost see Section 3.9 in their book.

values of  $\theta$  and  $\zeta$ , the government will simply maximize (3) under the restriction that (1) is non-negative:

$$\max_{I,P,T} \Omega(I, P, T | \theta, \zeta) \quad (4)$$

such that

$$\Pi(I, P, T | \theta, \zeta) \geq 0. \quad (5)$$

Without loss of generality it is assumed that under full information the outcome with no innovation ( $I = 0$ ) is never optimal. Let  $\{I^{FI}, P^{FI}, T^{FI}\}$  denote the solution to (4)–(5). The next proposition states that the choice between innovation policy instruments solely depends on the welfare loss parameters. The welfare loss for a dollar awarded as a prize is  $\lambda$  dollars. The loss for a dollar awarded through a patent is  $(1/\alpha - 1)$ : if the firm is to be awarded one additional dollar through an extended patent, i.e. if  $(\Delta\tau)\alpha\zeta w(I)/r = 1$ , the consumer will lose  $(\Delta\tau)\zeta w(I)/r = 1/\alpha$  dollars, so the net welfare cost of this dollar is  $(1/\alpha - 1)$ . A patent is therefore more efficient in awarding the dollar if and only if  $(1/\alpha - 1) < \lambda$ .<sup>12</sup> Having determined the optimal instrument, the optimal innovation size equates the marginal welfare of an additional unit of innovation to its marginal (innovation plus social) costs. The reward compensates the innovator for the innovation costs.

*Proposition 1. The optimal patent and prize system under full information satisfies:*

(1)

$$\frac{1 - \alpha}{\alpha} \geq \lambda \Leftrightarrow \begin{cases} T^{FI} = 0 \\ P^{FI} > 0 \\ \zeta w'(I^{FI}) = (1 + \lambda)r\theta c'(I^{FI}) \end{cases}$$

(2) If taxes can be used, then

$$\frac{1 - \alpha}{\alpha} < \lambda \Leftrightarrow \begin{cases} T^{FI} = \infty \\ P^{FI} < 0 \\ \zeta w'(I^{FI}) = \frac{1}{\alpha}r\theta c'(I^{FI}) \end{cases}$$

<sup>12</sup> I am grateful to an anonymous referee for suggesting this outline of the intuition of Proposition 1.

(3) If taxes are ruled out, then

$$\frac{1-\alpha}{\alpha} < \lambda \Leftrightarrow \begin{cases} T^{FI} > 0 \\ P^{FI} = 0 \\ \zeta w'(I^{FI}) = \frac{1}{\alpha} r \theta c'(I^{FI}) \end{cases}$$

(4)  $\Pi(I^{FI}, P^{FI}, T^{FI} | \theta, \zeta) = 0$ .

*Proof.* See the Appendix.

To see why these results hold, substitute for  $P$  and  $\tau$  from (1) into (3) to obtain the following expressions for welfare:

$$\begin{aligned} \Omega(I, P, T | \theta, \zeta) = & \{1 - \tau[1 - (1 + \lambda)\alpha]\} \frac{\zeta w(I)}{r} - (1 + \lambda)\theta c(I) \\ & - \lambda \Pi(I, P, T | \theta, \zeta) \end{aligned} \quad (6)$$

and

$$\begin{aligned} \Omega(I, P, T | \theta, \zeta) = & \frac{\zeta w(I)}{r} + \left[ \frac{1-\alpha}{\alpha} - \lambda \right] P - \frac{1}{\alpha} \theta c(I) \\ & - \frac{1-\alpha}{\alpha} \Pi(I, P, T | \theta, \zeta). \end{aligned} \quad (7)$$

Expressions (6)–(7) show that producer surplus exercises a negative influence on welfare, implying that it should be kept at its minimum, i.e. zero (result 4). They also indicate that the optimal patent length ( $\tau$ ) and prize ( $P$ ), depend on the sign of the expression in square brackets: if it is positive patents should be as short as possible (zero) and prizes as large as necessary to yield zero profits (result 1); if the expression is negative prizes should be as small as possible, i.e. zero when taxes are ruled out (with patents adjusted to yield zero profits; result 3) and sufficiently negative to yield zero profits in combination with an infinitely lived patent when taxes can be used (result 2). Note that the welfare loss parameters fully determine which of the two instruments is more efficient independent of  $\theta$  and  $\zeta$ .

The information asymmetries that are introduced in Sections 3 and 4 pertain to the firm's innovation cost and market size parameters,  $\theta$  and  $\zeta$ , respectively.<sup>13</sup> The firm knows the true value, but the government does not. It only has beliefs about the parameters, represented by probability density

<sup>13</sup> These parameters are equivalent to the multiplicative disturbances used in Wright (1983). Another possibility for modelling the asymmetric information regarding market properties is to introduce an uncertain market growth rate  $\gamma < r$ , so that the privately known effective discount rate would be  $\rho = r - \gamma$ . The main results in this paper do not depend on the type of asymmetric information about the market: market growth rate, market size or both.

functions  $f: [\underline{\theta}, \bar{\theta}] \rightarrow (0, \infty)$  and  $g: [\underline{\zeta}, \bar{\zeta}] \rightarrow (0, \infty)$  and the associated cumulative distribution functions  $F(\theta)$  and  $G(\zeta)$ . The domains of these density functions and the fact that they are strictly positive on the entire domain are common knowledge. The hazard rates  $\hat{f}(\theta)$  and  $\hat{g}(\zeta)$  of these distributions  $f(\cdot)$  and  $g(\cdot)$  are defined as:

$$\hat{f}(\theta) \equiv \frac{F(\theta)}{f(\theta)} \quad \hat{g}(\zeta) \equiv \frac{1 - G(\zeta)}{g(\zeta)}.$$

The following assumption regarding these hazard rates is not necessary to obtain the results presented in this paper, it merely simplifies the analysis considerably.

*Assumption 1 (monotone hazard rates).*  $\hat{f}(\theta)$  is a non-decreasing function of  $\theta$  and  $\hat{g}(\zeta)$  is a non-increasing function of  $\zeta$ .<sup>14</sup>

Under asymmetric information the government cannot announce a prize or patent system contingent on the unobservable information. The government has to design a mechanism defining a relationship between the firm's actions, the innovation size and the incentive reward. The revelation principle (see Myerson, 1979) states that whatever shape the optimal mechanism has, it can be equalled by a *direct* mechanism, which asks the firm to disclose its information and, contingent on this report, determines the innovation size and reward. This direct mechanism must be *incentive compatible*, i.e. designed in such a way that the firm has no incentive to misreport the value of the parameter.<sup>15</sup> Typically, a more informed agent can gain by lying, so that the government has to allow the innovator a strictly positive pay-off to induce this truth-telling behaviour. Since positive profits are sub-optimal ex-post these situations of asymmetric information generally entail welfare losses, i.e. they constitute a negative externality.

### 3. Asymmetric information about costs

This section investigates the optimal prizes and patents, when the value of the innovator's cost parameter  $\theta$  is private information: the innovator knows the true value, whereas the government only has beliefs, represented by the probability density function  $f(\theta)$ . Wherever this does not cause any ambiguity the commonly known market size parameter  $\zeta$  is omitted from

<sup>14</sup> This assumption is satisfied by many common distributions (see Laffont and Tirole, 1993, p. 66).

<sup>15</sup> The intuition behind the revelation principle is that the government can determine the firm's best reaction to the optimal mechanism and incorporate this reaction into the mechanism.

notation.

The revelation principle discussed above enables the government to restrict its attention to direct mechanisms that induce truthful reporting. To save notation define  $\Pi_T(\theta) \equiv \Pi(I(\theta), P(\theta), T(\theta)|\theta, \zeta)$  as the innovator's pay-off from reporting the truth  $\theta$ , and  $\Pi_L(\hat{\theta}|\theta) \equiv \Pi(I(\hat{\theta}), P(\hat{\theta}), T(\hat{\theta})|\theta, \zeta)$  as his pay-off from reporting the lie  $\hat{\theta}$ . The government has to solve the following problem.

$$\max_{I(\cdot), P(\cdot), T(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} \Omega(I(\theta), P(\theta), T(\theta)|\theta, \zeta) f(\theta) d\theta \tag{8}$$

such that

$$\Pi_T(\theta) \geq \Pi_L(\hat{\theta}|\theta), \quad \forall \theta, \hat{\theta} \in [\underline{\theta}, \bar{\theta}] \tag{9}$$

$$\Pi_T(\theta) \geq 0, \quad \forall \theta \in [\underline{\theta}, \bar{\theta}] \tag{10}$$

In this direct revelation problem, (9) is the incentive compatibility restriction (it should not pay to report the lie  $\hat{\theta}$  instead of the truth  $\theta$ ). The next lemma redefines problem (8)–(10).

*Lemma 1.* Let  $y_\lambda(\theta) \equiv (1 + \lambda)\theta + \lambda\hat{f}(\theta)$  and  $z_\alpha(\theta) \equiv \theta/\alpha + (1 - \alpha)\hat{f}(\theta)/\alpha$ . Then:

(1) Incentive compatibility condition (9) is satisfied if and only if

$$\theta > \hat{\theta} \Rightarrow I(\theta) \leq I(\hat{\theta}), \quad \forall \theta \in [\underline{\theta}, \bar{\theta}] \tag{11}$$

$$\Pi_T(\theta) = \Pi_T(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} c(I(\tilde{\theta})) d\tilde{\theta}, \quad \forall \theta \in [\underline{\theta}, \bar{\theta}] \tag{12}$$

(2) In the optimum  $\Pi_T(\bar{\tau}) = 0$ .

(3) The government's maximization problem (8) is then equivalent to both<sup>16</sup>

$$\int_{\underline{\theta}}^{\bar{\theta}} \left\{ [1 - \tau(\theta)][1 - (1 + \lambda)\alpha] \right\} \frac{\zeta w(I(\theta))}{r} - y_\lambda(\theta) c(I(\theta)) \Big\} f(\theta) d\theta \tag{13}$$

and

<sup>16</sup> Remember that  $T(\theta)$  enters via  $\tau(\theta)$ .

$$\int_{\underline{\theta}}^{\bar{\theta}} \left\{ \frac{\xi w(I(\theta))}{r} + \left[ \frac{1-\alpha}{\alpha} - \lambda \right] P(\theta) - z_{\alpha}(\theta)c(I(\theta)) \right\} f(\theta) d\theta . \tag{14}$$

*Proof.* See the Appendix.

Restrictions like (11)–(12) can be derived often in regulation problems. To see why they are necessary for incentive compatibility observe that the pay-off from lying is  $\pi_L(\hat{\theta}|\theta) = \Pi_T(\hat{\theta}) + (\hat{\theta} - \theta)c(I(\hat{\theta}))$ , i.e. a liar obtains the truthful report pay-off of the type he pretends to be plus his cost advantage over this type. This means that the difference between the truthful report pay-offs of any two types must be bounded by the cost (dis)advantage that each of these types has when pretending to be the other:

$$(\hat{\theta} - \theta)c(I(\hat{\theta})) \leq \Pi_T(\theta) - \Pi_T(\hat{\theta}) \leq (\hat{\theta} - \theta)c(I(\theta)) . \tag{15}$$

Monotonicity condition (11) follows directly from (15). A limiting argument ( $\hat{\theta} \rightarrow \theta$ ) implies that incentive compatibility should be locally binding: the gain from an infinitesimal deviation from the truth should be exactly offset by its cost, i.e.  $\Pi_T(\theta) - \Pi_T(\theta - d\theta) = -c(I(\theta))d\theta$ . Condition (12) follows from integration of this equality.

Together with  $\Pi_T(\bar{\theta}) \geq 0$  restriction (12) guarantees (10). This inequality is binding in the optimum (result 2) because excessive profits for the innovator are socially costly.

Lemma 1 redefines the government’s problem as maximization of (13) and/or (14) subject to (11) and

$$\Pi_T(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} c(I(\tilde{\theta})) d\tilde{\theta} \quad \forall \theta \in [\underline{\theta}, \bar{\theta}] . \tag{16}$$

The integrands in (13) and (14) are very similar to the full information maximands (6) and (7), respectively. More importantly, the hazard rate—representing the element of uncertainty—does not enter the trade-off between patents ( $\tau(\theta) > 0$ ) and prizes ( $\tau(\theta) = 0$ ). Analogously to the full information case, if the expression in square brackets is negative (positive), the prize (duration of the patent) should be as small as possible and the duration of the patent (prize) as large as necessary to satisfy (16).

Given the optimal patent duration and prize for each  $\theta$ , the integrand in (13) and/or (14) can be maximized for each  $\theta$  to give the optimal innovation sizes. Due to the monotone hazard rate assumption  $y_{\lambda}(\theta)$  and  $z_{\alpha}(\theta)$  are increasing in  $\theta$ , which ensures that these innovation sizes satisfy the monotonicity condition (11). Since (16) dictates that profits to an innovator type have to be given to all smaller types as well and since giving these

profits is socially costly, the government wants to reduce  $\Pi_T(\cdot)$  by inducing the firm to perform suboptimal innovations (relative to the full information case). Hence,  $y_\lambda(\theta) > (1 + \lambda)\theta$  and  $z_\alpha(\theta) > \theta/\alpha$  for all but the smallest  $\theta$ . The monotonicity of  $y_\lambda(\theta)$  and  $z_\alpha(\theta)$  suggests that it is desirable to reduce the size of the innovation more for high- $\theta$  innovators.

Proposition 2 states the entire solution to the optimal prize and patent system problem under asymmetric cost (AC) information represented by (8)-(10). This solution is denoted by  $\{I^{AC}(\theta), P^{AC}(\theta), T^{AC}(\theta)\}$ .<sup>17</sup>

*Proposition 2. Let  $y_\lambda(\theta)$  and  $z_\alpha(\theta)$  be defined as in Lemma 1. The optimal patent and prize system under asymmetric information about the innovation costs satisfies:*

(1)

$$\frac{1 - \alpha}{\alpha} \geq \lambda \Leftrightarrow \begin{cases} T^{AC}(\theta) \equiv 0 \\ P^{AC}(\theta) \geq 0 & \forall \theta \in [\underline{\theta}, \bar{\theta}] \\ I^{AC}(\theta) \in \operatorname{argmax}_I \left\{ \frac{\zeta w(I)}{r} - y_\lambda(\theta)c(I) \right\} & \forall \theta \in [\underline{\theta}, \bar{\theta}] \end{cases}$$

(2) *If taxes can be used, then*

$$\frac{1 - \alpha}{\alpha} < \lambda \Leftrightarrow \begin{cases} T^{AC}(\theta) \equiv \infty \\ P^{AC}(\theta) \leq 0 & \forall \theta \in [\underline{\theta}, \bar{\theta}] \\ I^{AC}(\theta) \in \operatorname{argmax}_I \left\{ \alpha(1 + \lambda) \frac{\zeta w(I)}{r} - y_\lambda(\theta)c(I) \right\} & \forall \theta \in [\underline{\theta}, \bar{\theta}] \end{cases}$$

(3) *If taxes are ruled out, then*

$$\frac{1 - \alpha}{\alpha} < \lambda \Leftrightarrow \begin{cases} T^{AC}(\theta) \geq 0 & \forall \theta \in [\underline{\theta}, \bar{\theta}] \\ P^{AC}(\theta) \equiv 0 \\ I^{AC}(\theta) \in \operatorname{argmax}_I \left\{ \frac{\zeta w(I)}{r} - z_\alpha(\theta)c(I) \right\} & \forall \theta \in [\underline{\theta}, \bar{\theta}] \end{cases}$$

(4)

$$\Pi(I^{AC}(\theta), P^{AC}(\theta), T^{AC}(\theta)|\theta, \zeta) = \int_{\theta}^{\bar{\theta}} c(I^{AC}(\tilde{\theta}))d\tilde{\theta} \quad \forall \theta \in [\underline{\theta}, \bar{\theta}].$$

<sup>17</sup> This direct mechanism has a non-direct equivalent in which patents and prizes are contingent on the innovation size; it can be derived in a straightforward way from the characterization in Proposition 2.

*Proof.* See the Appendix.

From Propositions 1 and 2 the next theorem follows directly. It says that the answer to the question of which instrument is more efficient does not depend on the information structure regarding the costs to innovation. Although asymmetric cost information entails welfare losses, they are the same for prizes and patents, leaving the trade-off between the two instruments unchanged.

*Theorem 1. A patent (prize) is the more efficient instrument under asymmetric information about innovation costs if and only if a patent (prize) is the more efficient instrument under full information, irrespective of the value of the cost parameter.*

#### 4. Asymmetric information about the market

Having also found that asymmetric cost information does not affect the choice between patents and prizes, Wright (1983) suggests that asymmetric information about the market may influence the relative performance of patents. Since patents give the reward through exploitation of the innovation in the market, a patent system may be more efficient in the provision of incentives to disclose the information, which would mitigate the information externality.

In this section the value of the market size parameter  $\zeta$  is private information: the innovator knows the true value, the government only has beliefs, represented by the probability density function  $g(\zeta)$ . The commonly known cost parameter  $\theta$  is omitted from notation wherever this does not cause any ambiguity.

Redefining the innovator's pay-offs from reporting the truth ( $\zeta$ ) and a lie ( $\hat{\zeta}$ ), as  $\Pi_T(\zeta) \equiv \Pi(I(\zeta), P(\zeta), T(\zeta)|\theta, \zeta)$  and  $\Pi_L(\hat{\zeta}|\zeta) \equiv \Pi(I(\hat{\zeta}), P(\hat{\zeta}), T(\hat{\zeta})|\theta, \zeta)$ , respectively, the direct revelation mechanism that the government considers is similar to (8)-(10):

$$\max_{I(\cdot), P(\cdot), T(\cdot)} \int_{\underline{\zeta}}^{\bar{\zeta}} \Omega(I(\zeta), P(\zeta), T(\zeta)|\theta, \zeta) g(\zeta) d\zeta \quad (17)$$

such that

$$\Pi_T(\zeta) \geq \Pi_L(\hat{\zeta}|\zeta), \quad \forall \zeta, \hat{\zeta} \in [\underline{\zeta}, \bar{\zeta}] \quad (18)$$

$$\Pi_T(\zeta) \geq 0, \quad \forall \zeta \in [\underline{\zeta}, \bar{\zeta}] \quad (19)$$

The redefinition of this problem given in the next lemma resembles (11)–(14).

*Lemma 2.* Let  $x_{\alpha\lambda}(\zeta) \equiv 1 - \alpha - \alpha\lambda(1 - \hat{g}(\zeta)/\zeta)$ .

(1) A set of necessary and sufficient conditions for (18) is given by

$$\zeta > \hat{\zeta} \Rightarrow \tau(\zeta) \cdot w(I(\zeta)) \geq \tau(\hat{\zeta}) \cdot w(I(\hat{\zeta})), \quad \forall \zeta, \hat{\zeta} \in [\underline{\zeta}, \bar{\zeta}] \quad (20)$$

$$\Pi_T(\zeta) = \Pi_T(\underline{\zeta}) + \int_{\underline{\zeta}}^{\zeta} \alpha \tau(\tilde{\zeta}) \frac{w(I(\tilde{\zeta}))}{r} d\tilde{\zeta}, \quad \forall \zeta \in [\underline{\zeta}, \bar{\zeta}] \quad (21)$$

(2) In the optimum  $\Pi_T(\underline{\zeta}) = 0$ .

(3) The government's maximization problem (17) is then equivalent to<sup>18</sup>

$$\int_{\underline{\zeta}}^{\bar{\zeta}} \left\{ \left[ 1 - \tau(\zeta) \cdot x_{\alpha\lambda}(\zeta) \right] \frac{\zeta w(I(\zeta))}{r} - (1 + \lambda)\theta c(I(\zeta)) \right\} g(\zeta) d\zeta. \quad (22)$$

*Proof.* Omitted (completely analogous to the proof of Lemma 1).

From (22) it is clear that the best instrument to use is determined by the sign of  $x_{\alpha\lambda}(\zeta)$ . In contrast to the case of asymmetric information about the innovation costs, here the uncertainty does enter the trade-off between patents and prizes. Since the sign of  $x_{\alpha\lambda}(\zeta)$  is contingent on  $\zeta$  the optimal instrument may differ across market size values. More specifically, since  $x_{\alpha\lambda}(\zeta)$  is non-increasing it may be positive (prizes are better) for small market sizes and negative (patents are better) for large ones.

Let the solution to the optimal prize and patent system problem under asymmetric market (AM) information given by (17)–(19), be denoted by  $\{I^{\text{AM}}(\zeta), P^{\text{AM}}(\zeta), T^{\text{AM}}(\zeta)\}$ . Proposition 3 characterizes this solution.

*Proposition 3.* Let

$$q_{\alpha}(\zeta) \equiv \frac{1}{\alpha} + \frac{1 - \alpha}{\alpha \zeta^2 g(\zeta)} \int_{\underline{\zeta}}^{\bar{\zeta}} \tilde{\zeta} g(\tilde{\zeta}) d\tilde{\zeta}$$

<sup>18</sup> A second alternative to (17) cannot be derived, since substituting for  $\tau(\zeta)$  from (21) is not possible.

be non-increasing in  $\zeta$ . The optimal patent and prize system under asymmetric information about the market size satisfies:<sup>19</sup>

(1)

$$\frac{1-\alpha}{\alpha} \geq \lambda \left(1 - \frac{\hat{g}(\zeta)}{\zeta}\right) \Leftrightarrow \begin{cases} T^{AM}(\zeta) \equiv 0 \\ P^{AM}(\zeta) \geq 0 & \forall \zeta \in [\underline{\zeta}, \bar{\zeta}] \\ I^{AM}(\zeta) \in \operatorname{argmax}_I \left\{ \frac{\zeta w(I)}{r} - (1+\lambda)\theta c(I) \right\} & \forall \zeta \in [\underline{\zeta}, \bar{\zeta}] \end{cases}$$

(2) If taxes can be used, then

$$\frac{1-\alpha}{\alpha} < \lambda \left(1 - \frac{\hat{g}(\zeta)}{\zeta}\right) \Leftrightarrow \begin{cases} T^{AM}(\zeta) \equiv \infty \\ P^{AM}(\zeta) \leq 0 & \forall \zeta \in [\underline{\zeta}, \bar{\zeta}] \\ I^{AM}(\zeta) \in \operatorname{argmax}_I \left\{ \frac{\zeta w(I)}{r} \left[1 - \frac{\lambda}{1+\lambda} \frac{\hat{g}(\zeta)}{\zeta}\right] - \frac{1}{\alpha} \theta c(I) \right\} & \forall \zeta \in [\underline{\zeta}, \bar{\zeta}] \end{cases}$$

(3) If taxes are ruled out, then

$$\frac{1-\alpha}{\alpha} < \lambda \left(1 - \frac{\hat{g}(\zeta)}{\zeta}\right) \Leftrightarrow \begin{cases} T^{AM}(\zeta) \geq 0 & \forall \zeta \in [\underline{\zeta}, \bar{\zeta}] \\ P^{AM}(\zeta) \equiv 0 \\ I^{AM}(\zeta) \in \operatorname{argmax}_I \left\{ \frac{\zeta w(I)}{r} - q_a(\zeta)\theta c(I) \right\} & \forall \zeta \in [\underline{\zeta}, \bar{\zeta}] \end{cases}$$

(4)

$$\Pi(I^{AM}(\zeta), P^{AM}(\zeta), T^{AM}(\zeta) | \theta, \zeta) = \int_{\underline{\zeta}}^{\zeta} \alpha \tau^{AM}(\tilde{\zeta} | \theta) \frac{w(I^{AM}(\tilde{\zeta}))}{r} d\tilde{\zeta} \quad \forall \zeta \in [\underline{\zeta}, \bar{\zeta}].$$

*Proof.* See the Appendix.

<sup>19</sup> The restrictiveness of the condition on  $q_a(\zeta)$  is comparable to that of the monotone hazard rate assumptions. Again, this condition is only needed to allow a representation of the solution, not for the main results of the paper.

Surprisingly, prizes are preferred more often under market uncertainty than under full information: since the hazard rate is non-negative, case 1 of Proposition 3 will occur more often than case 1 of Proposition 1. Specifically, if under full information prizes are better then under market uncertainty they are better as well, but if under full information patents are preferred then under market uncertainty prizes may be preferred for small market sizes (since  $\hat{g}(\bar{\zeta}) = 0$ , high types will still be awarded a patent).

To explain this result, note that the innovations performed when prizes are used (case 1) are identical to the ones under full information, and that the innovator's net profits equal zero in this case. Thus, there is no information externality if prizes are used in the case of market uncertainty: the innovator cannot use his private information strategically to influence the amount of prize money, since the prize always equals the (observable) innovation costs independent of the innovator's report. Remember that under cost uncertainty the welfare costs associated with patents and prizes had increased in the same way due to the information externality, so that the trade-off between the instruments was left unchanged. In contrast to this, when the private information pertains to the market the externality only applies to patents, not to prizes, implying that the trade-off must have changed in favour of prizes.

The results above are direct implications of Propositions 1 and 3 and are stated formally in the next theorem.

*Theorem 2. If under full information a prize is the more efficient instrument, then under asymmetric information about the market a prize is the more efficient instrument for all market size parameter values; but if under full information a patent is the more efficient instrument, then under asymmetric information about the market a combined prize and patent system may be more efficient than a pure patent system.*

## 5. Concluding remarks

This paper compares the welfare properties of research prizes and patents for the provision of innovation incentives in a monopolistic R&D process. It investigates how their performance is related to information asymmetries between the government and the innovating firm regarding the firm's innovation costs and the market size, respectively. Two results are obtained: (i) asymmetric information about the innovation costs does not change the trade-off between patents and prizes; and (ii) asymmetric information about the market for the innovation may induce the government to switch from patents to prizes for small innovations, but never from prizes to patents.

These results are due to the fact that cost uncertainty exerts the same negative externality on the welfare properties of the two instruments, whereas market uncertainty only exerts a negative externality on the properties of patents.

Two remarks have to be made regarding these results. First, one may wonder what happens in the case of asymmetric information about both costs and the market; an answer to this question cannot be obtained in the context of this model. Second, the size of the welfare loss associated with prizes can be expected to be positively correlated with their frequency and amount of use. An increase in the use of prizes is likely to result in a larger shadow cost of public funds, since more funds have to be raised.

Together with the work of Wright (1983) this paper covers the area of information asymmetries as a potential justification for the use of patents. The results can only be characterized as inconclusive. Thus, the search for a more convincing justification must be directed along other paths, some of which are mentioned below.

Wright only considers one of the socially undesirable aspects of competition in R&D: the common pool problem. Several other inefficiency results regarding R&D competition exist: competitors prefer risky R&D projects to safe ones, in some situations the research paths they choose are too correlated and R&D competition may result in a pace of technological progress that exceeds the socially optimal speed.<sup>20</sup> Further research should examine whether a patent system is better equipped to alleviate these inefficiencies.

A rationale for patents may also be contained in their flexibility. The reward offered by a patent not only depends on the duration, but also on its protectiveness against imitators (see, for example, Gilbert and Shapiro, 1990; Klemperer, 1990) and the minimum improvement requirement that an innovation must meet to earn a new patent (Scotchmer and Green, 1990). Compared to this multi-dimension property of patents, a prize is a rather rigid instrument.

Another research path takes into consideration that innovators often do not have to rely entirely on patents to regain their R&D investments in the market. Industry appropriability conditions and the technological lead can give the innovator market power due to lagged and/or imperfect imitation. This phenomenon (especially when its magnitude is private information) may influence the welfare performance of patents and prizes.

<sup>20</sup> See, for example, Dasgupta and Stiglitz (1980), Bhattacharya and Mookherjee (1986), Dasgupta and Maskin (1987) and Olsen (1993).

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## Appendix

### A1. Proof of Proposition 1

From (1) and (3) it is clear that  $\Omega(I, P, T|\theta, \zeta)$  decreases and  $\Pi(I, P, T|\theta, \zeta)$  increases with  $P$  and  $T$ , so that if (5) is not satisfied with equality—implying positive  $P$  and/or  $T$ —,  $P$  and/or  $T$  can be reduced until  $\Pi(I, P, T|\theta, \zeta) = 0$ , which will always increase welfare. Thus, (5) will always be satisfied with equality; this is result 4 in the proposition.

Maximands equivalent to (4) can be obtained by substituting for either  $P$  or  $\tau$  from  $\Pi(I, P, T|\theta, \zeta) = 0$  into (4). The problem can then be restated as

$$\max_{I, P, T} \frac{\zeta w(I)}{r} - \left[ \lambda - \frac{1 - \alpha}{\alpha} \right] P - \frac{1}{\alpha} \theta c(I) \quad \text{s.t.} \quad \tau = \frac{r[\theta c(I) - P]}{\alpha \zeta w(I)} \quad (\text{A.1})$$

or, equivalently,

$$\begin{aligned} \max_{I, P, T} \frac{\zeta w(I)}{r} \{1 - \tau[1 - \alpha(1 + \lambda)]\} - (1 + \lambda)\theta c(I) \quad \text{s.t.} \quad P = \theta c(I) \\ - \tau \alpha \frac{\zeta w(I)}{r}. \end{aligned} \quad (\text{A.2})$$

From (A.1), if  $\lambda - (1 - \alpha)/\alpha > 0$ ,  $P$  should be as small as possible. If taxes are not allowed, this means  $P^{\text{FI}} = 0$ , so that

$$I^{\text{FI}} \in \operatorname{argmax}_I \left\{ \frac{\zeta w(I)}{r} - \frac{1}{\alpha} \theta c(I) \right\} \quad (\text{A.3})$$

and

$$\tau^{\text{FI}} = \frac{r\theta c(I^{\text{FI}})}{\alpha \zeta w(I^{\text{FI}})} > 0.$$

This is result 3 in the proposition.

If taxes can be used,  $P$  as small as possible implies that patent duration is at its upper bound (infinity), i.e.  $\tau^{\text{FI}} = 1$ . Using this in (A.1) we get

$P^{FI} = \theta c(I^{FI}) - \alpha \zeta w(I^{FI})/r$ , which we can substitute in the maximand to obtain

$$I^{FI} \in \operatorname{argmax}_I \left\{ \alpha(1 + \lambda) \frac{\zeta w(I)}{r} - (1 + \lambda)\theta c(I) \right\}. \tag{A.4}$$

Since (A.4) is positive in the optimum,  $P^{FI} < 0$ . This is result 2.

From (A.2), if  $1 - \alpha(1 + \lambda) > 0$ —which is equivalent to  $\lambda < (1 - \alpha)/\alpha - \tau$ —should be as small as possible, i.e.  $\tau^{FI} = T^{FI} = 0$ , so that

$$I^{FI} \in \operatorname{argmax}_I \left\{ \frac{\zeta w(I)}{r} - (1 + \lambda)\theta c(I) \right\} \tag{A.5}$$

and  $P^{FI} = \theta c(I^{FI}) > 0$ .

If  $\lambda = (1 - \alpha)/\alpha$ , the maximization problems (A.3), (A.4), (A.5) are equivalent and prizes and patents yield the same welfare level. By assumption then the government prefers to use prizes:  $T^{FI} = 0$ ,  $P^{FI} = \theta c(I^{FI}) > 0$ . This completes the proof of Proposition 1. Q.E.D.

*A2. Proof of Lemma 1*

The proof of result 1 is quite standard in the regulation literature (see, for example, Proposition 1.2 in Laffont and Tirole, 1993, p. 64) and is omitted.

Suppose  $\{I_1(\cdot), P_1(\cdot), T_1(\cdot)\}$  solves the problem and  $\Pi(I_1(\bar{\theta}), P_1(\bar{\theta}), T_1(\bar{\theta})|\bar{\theta}, \zeta) = \Pi_1 > 0$ . Then an alternative solution  $\{I_1(\cdot), P_1(\cdot), T_1(\cdot)\}$  with  $\Pi(I_1(\theta), P_2(\theta), T_2(\theta)|\theta, \zeta) = \Pi(I_1(\theta), P_2(\theta), T_2(\theta)|\theta, \zeta) - \Pi_1$  can be constructed that satisfies the constraints and increases welfare. Thus,  $\{I_1(\cdot), P_1(\cdot), T_1(\cdot)\}$  cannot have been a solution and, therefore, a solution must satisfy  $\Pi(I(\bar{\theta}), P(\bar{\theta}), T(\bar{\theta})|\bar{\theta}, \zeta) = 0$ . This completes the proof of result 2.

Maximand (13) is derived by substituting for  $P(\theta)$  from (12) into (8) using result 2 and

$$\begin{aligned} \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} c(I(\tilde{\theta})) d\tilde{\theta} f(\theta) d\theta &= \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\bar{\theta}} f(\theta) d\theta c(I(\tilde{\theta})) d\tilde{\theta} = \int_{\underline{\theta}}^{\bar{\theta}} F(\tilde{\theta})c(I(\tilde{\theta})) d\tilde{\theta} \\ &= \int_{\underline{\theta}}^{\bar{\theta}} \hat{f}(\theta)c(I(\theta))f(\theta) d\theta. \end{aligned}$$

(14) is derived analogously by substituting for  $\tau(\theta)$ . This completes the proof of Lemma 1. Q.E.D.

### A3. Proof of Proposition 2

The proof is analogous to the proof of Proposition 1, except (i) the proposed solution satisfies the monotonicity condition (11), and (ii)  $\tau^{AC} \leq 1$ ,  $\forall \theta \in [\underline{\theta}, \bar{\theta}]$ . The monotonicity of the solution is a consequence of the monotonicity of  $y_\lambda(\theta)$  and  $z_\alpha(\theta)$ . First, note that for large values of  $\theta$ , the maximization problems that define the optimal innovation size may render the border solution  $I=0$ . For small (possibly all) values of  $\theta$  these maximization problems yield positive innovation sizes, defined by their first order conditions:

$$\frac{\zeta w'(I^{AC}(\theta))}{rc'(I^{AC}(\theta))} = \begin{cases} y_\lambda(\theta) & \text{for case 1} \\ y_\lambda(\theta) & \text{for case 2} \\ z_\alpha(\theta) & \text{for case 3.} \end{cases}$$

The left hand side of this expression is a non-increasing function of  $I$ , since  $w(\cdot)$  is a concave and  $c(\cdot)$  a convex function. Together with the fact that for all three cases the right hand side is increasing in  $\theta$ , this means that these first order conditions yield optimal innovation sizes that are decreasing in  $\theta$ . This proves (i).

For cases 1 and 2, (ii) is trivial. For case 3 and for parameter values with  $I^{AC}(\theta) > 0$ :

$$\begin{aligned} \tau^{AC}(\theta) &= \frac{r\theta c(I^{AC}(\theta)) + r \int_{\theta}^{\bar{\theta}} c(I^{AC}(\tilde{\theta})) d\tilde{\theta}}{\alpha \zeta w(I^{AC}(\theta))} \\ &= \frac{r\bar{\theta} c(I^{AC}(\bar{\theta})) - r \int_{\theta}^{\bar{\theta}} \bar{\theta} dc(I^{AC}(\tilde{\theta}))}{\alpha \zeta w(I^{AC}(\theta))} \\ &< \frac{r\bar{\theta} c(I^{AC}(\bar{\theta})) - \alpha \int_{\theta}^{\bar{\theta}} r z_\alpha(\tilde{\theta}) dc(I^{AC}(\tilde{\theta}))}{\alpha \zeta w(I^{AC}(\theta))} \\ &= \frac{r\bar{\theta} c(I^{AC}(\bar{\theta})) - \alpha \zeta \int_{\theta}^{\bar{\theta}} dw(I^{AC}(\tilde{\theta}))}{\alpha \zeta w(I^{AC}(\theta))} \end{aligned}$$

$$= \frac{\alpha \zeta w(I^{AC}(\theta)) - [\alpha \zeta w(I^{AC}(\bar{\theta})) - r \bar{\theta} c(I^{AC}(\bar{\theta}))]}{\alpha \zeta w(I^{AC}(\theta))} \leq 1.$$

The second equality uses integration by parts, the first inequality uses  $\alpha z_c(\theta) > \theta$  for  $\theta > \bar{\theta}$  and the third equality uses the first order condition for  $I^{AC}(\theta)$ . This completes the proof of Proposition 2. Q.E.D.

A4. Proof of Proposition 3

Similar to the proof of Proposition 1, it is clear that the sign of  $x_{\alpha}(\zeta)$  determines whether  $\tau(\zeta)$  should be as large or as small as possible. Since  $\hat{g}(\zeta)/\zeta$  is non-increasing (because  $\hat{g}(\zeta)$  is non-increasing), only three situations can occur: (i) case 1 prevails for all  $\zeta$ , (ii) case 2 or 3 prevails for all  $\zeta$ , or (iii) case 1 prevails for small values of  $\zeta$  and case 2 or 3 for large values. The solutions proposed for cases 1 and 2 follow the same line of reasoning as Proposition 2. If case 3 ever prevails, this happens in situation (ii) or (iii) so that it prevails along the entire interval  $(\zeta_0, \bar{\zeta}]$  for some  $\zeta_0 \geq \underline{\zeta}$ ; the problem to solve then can be written as:

$$\max_{I(\zeta), T(\zeta)} \int_{\zeta_0}^{\bar{\zeta}} \left\{ \frac{\zeta w(I(\zeta))}{r} - \tau(\zeta)(1 - \alpha) \frac{\zeta w(I(\zeta))}{r} - \theta c(I(\zeta)) \right\} g(\zeta) d\zeta \tag{A.6}$$

such that

$$\tau(\zeta) \alpha \frac{\zeta w(I(\zeta))}{r} - \theta c(I(\zeta)) = \int_{\zeta_0}^{\zeta} \tau(\tilde{\zeta}) \alpha \frac{w(I(\tilde{\zeta}))}{r} d\tilde{\zeta} \quad \forall \zeta \in (\zeta_0, \bar{\zeta}] \tag{A.7}$$

$$\zeta > \hat{\zeta} \Rightarrow \tau(\zeta) w(I(\zeta)) \geq \tau(\hat{\zeta}) w(I(\hat{\zeta})) \quad \forall \zeta, \hat{\zeta} \in (\zeta_0, \bar{\zeta}]. \tag{A.8}$$

Introducing a state variable

$$V(\zeta) \equiv \tau(\zeta) \alpha \frac{\zeta w(I(\zeta))}{r} - \theta c(I(\zeta)),$$

problem (A.6)–(A.7) can be rewritten as an optimal control problem:

$$\max_{I(\zeta)} \int_{\zeta_0}^{\bar{\zeta}} \left\{ \frac{\zeta w(I(\zeta))}{r} - \frac{1}{\alpha} \theta c(I(\zeta)) - \frac{1 - \alpha}{\alpha} V(\zeta) \right\} g(\zeta) d\zeta \tag{A.9}$$

such that

$$\dot{V}(\zeta) = \frac{V(\zeta) + \theta c(I(\zeta))}{\zeta} \quad \forall \zeta \in (\zeta_0, \bar{\zeta}] \tag{A.10}$$

$$V(\zeta_0) = 0. \tag{A.11}$$

The Hamiltonian of this problem reads

$$H(I, V, p_0, p_1, \zeta) = p_0 \left[ \frac{\zeta w(I)}{r} - \frac{1}{\alpha} \theta c(I) - \frac{1-\alpha}{\alpha} V \right] g(\zeta) + p_1 \frac{V + \theta c(I)}{\zeta}.$$

The solution to (A.9)–(A.11) is then given by the maximum principle:

$$(p_0^*, p_1^*(\zeta)) \neq (0, 0) \quad \forall \zeta \in (\zeta_0, \bar{\zeta}] \tag{A.12}$$

$$-H_V = p_0^* \frac{1-\alpha}{\alpha} g(\zeta) - \frac{p_1^*(\zeta)}{\zeta} = \dot{p}_1^*(\zeta) \quad \forall \zeta \in (\zeta_0, \bar{\zeta}] \tag{A.13}$$

$$p_1^*(\bar{\zeta}) = 0 \tag{A.14}$$

$$I^*(\zeta) \in \operatorname{argmax}_I H(I, V(\zeta), p_0^*, p_1^*(\zeta), \zeta) \quad \forall \zeta \in (\zeta_0, \bar{\zeta}]. \tag{A.15}$$

From (A.12) and (A.14) it follows immediately that  $p_0^* \neq 0$ ; we then normalize  $p_0^* = 1$ . Solving (A.13) and taking into account boundary condition (A.14) gives

$$p_1^*(\zeta) = -\frac{1-\alpha}{\alpha \zeta} \int_{\zeta}^{\bar{\zeta}} \tilde{\zeta} g(\tilde{\zeta}) d\tilde{\zeta}.$$

Using this, (A.15) reduces to

$$I^*(\zeta) \in \operatorname{argmax}_I \left\{ \frac{\zeta w(I)}{r} - q_\alpha(\zeta) \theta c(I) \right\}.$$

Now it is left to show that, if  $T^*(\cdot)$  satisfies (A.7) using  $I^*(\cdot)$ ,  $I^*(\cdot)$  and  $T^*(\cdot)$  solve (A.6)–(A.8). To see this, first observe that if  $q_\alpha(\zeta)$  is non-increasing  $I^*(\zeta)$  is non-decreasing in  $\zeta$ . Furthermore,  $I(\cdot)$  non-decreasing is necessary and sufficient for (A.8) if (A.7) holds:

$$\begin{aligned} \theta c(I(\zeta_2)) - \theta c(I(\zeta_1)) &= \frac{\alpha}{r} \left[ \tau(\zeta_2) \zeta_2 w(I(\zeta_2)) - \tau(\zeta_1) \zeta_1 w(I(\zeta_1)) \right. \\ &\quad \left. - \int_{\zeta_1}^{\zeta_2} \tau(\tilde{\zeta}) w(I(\tilde{\zeta})) d\tilde{\zeta} \right] \\ &= \frac{\alpha}{r} \int_{\zeta_1}^{\zeta_2} \tilde{\zeta} d\tau(\tilde{\zeta}) w(I(\tilde{\zeta})), \end{aligned}$$

where the second step uses integration by parts. Feasibility of  $T^*(\zeta)$  can be shown analogously to Proposition 2. Thus, it has been shown that  $I^*(\cdot)$ ,  $T^*(\cdot)$  solve case 3 if  $q_\alpha(\zeta)$  is non-increasing in  $\zeta$ . This completes the proof of Proposition 3. Q.E.D.

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